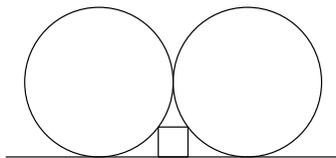


24. A square fits snugly between the horizontal line and two touching circles of radius 1. What is its side length?

A) $\frac{2}{5}$ B) $\frac{1}{4}$ C) $\frac{1}{\sqrt{2}}$ D) $\frac{1}{5}$ E) $\frac{1}{2}$



25. Tom wants to write several distinct positive integers, none of them exceeding 100. Their product should not be divisible by 54. At most how many integers can he write?

A) 8 B) 17 C) 68 D) 69 E) 90

26. Two regular polygons of side length 1 lie on opposite sides of their common side AB . One of them is a 15-gon $ABCD \dots$ and the other is an n -gon $ABZY \dots$. What value of n makes the distance CZ equal to 1?

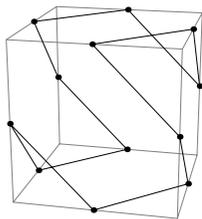
A) 10 B) 12 C) 15 D) 16 E) 18

27. The equalities $k = \sqrt[3]{2014+m} = \sqrt[3]{1024} + 1$ are given for positive integers k, m, n . How many different values can the number m take?

A) None B) 1 C) 2 D) 3 E) Infinitely many

28. The diagram shows a polygon whose vertices are the mid-points of the edges of a cube. An interior angle of the polygon is defined in the normal way: the angle between the two edges meeting at a vertex. What is the sum of all the interior angles of the polygon?

A) 720° B) 1080° C) 1200° D) 1440° E) 1800°

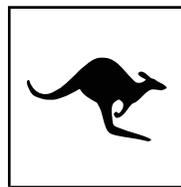


29. The function $f: \mathbb{Z} \rightarrow \mathbb{Z}$ satisfies the conditions $f(4) = 6$ and $xf(x) = (x-3)f(x+1)$. What is the value of $f(4)f(7)f(10) \dots f(2011)f(2014)$?

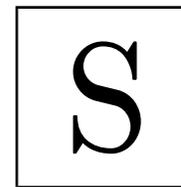
A) 2013 B) 2014 C) $2013 \cdot 2014$ D) $2013!$ E) 2014!

30. In the forests of a magical island three kinds of animals roam: lions, wolves and goats. Wolves can eat goats, and lions can eat either wolves or goats. However, this being a magical island: If a wolf eats a goat, it turns into a lion. If a lion eats a goat, it turns into a wolf. If a lion eats a wolf, it turns into a goat. Originally, there were 17 goats, 55 wolves and 6 lions on the island. What is the highest possible number of animals remaining on the island after no more eating is possible to happen?

A) 1 B) 6 C) 17 D) 23 E) 35



KANGAROO 2014



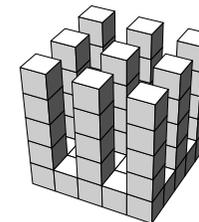
Time allowed: 75 minutes
Calculators are not permitted

Student
11–12 grades

Questions for 3 points

1. If you take a number of $1 \times 1 \times 1$ cubes out of a $5 \times 5 \times 5$ cube, you end up with a solid figure consisting of columns of the same height, which stand on the same ground plate (see figure). How many small cubes were taken out?

A) 56 B) 60 C) 64 D) 68 E) 80



2. Today is Carla's, Emilie's and Lilia's birthday. The sum of their ages is now 44. What will the sum of their ages be the next time it is a two-digit number with two equal digits?

A) 55 B) 66 C) 77 D) 88 E) 99

3. If $a^b = \frac{1}{2}$ what is the value of a^{-3b} ?

A) $\frac{1}{8}$ B) 8 C) -8 D) 6 E) $\frac{1}{6}$

4. There are 48 balls placed into three baskets of different sizes. The smallest and the largest basket contain together twice the number of balls that the middle one contains. The smallest basket contains half the number of balls of the middle one. How many balls are there in the largest basket?

A) 16 B) 20 C) 24 D) 30 E) 32

5. Find $\frac{2^{2014} - 2^{2013}}{2^{2013} - 2^{2012}}$.

A) 2^{2011} B) 2^{2012} C) 2^{2013} D) 1 E) 2

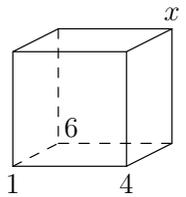
6. What is the negation of the statement "Everybody solved more than 20 problems"?

A) Nobody solved more than 20 problems B) Somebody solved less than 21 problems
C) Everybody solved less than 21 problems D) Somebody solved exactly 20 problems
E) Somebody solved more than 20 problems

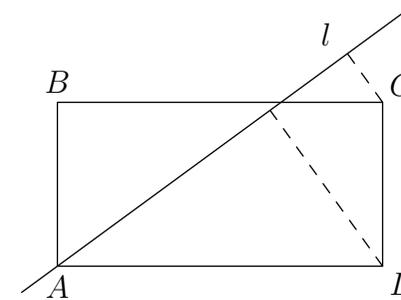
7. How many digits long is the result of the multiplication: $(2^{22})^5 \cdot (5^{55})^2$?

A) 22 B) 55 C) 77 D) 110 E) 111

8. In a coordinate system Tom drew a square. The coordinates of the two opposite vertices on the x -axis are $(-1; 0)$ and $(5; 0)$. Which of the following are the coordinates of another vertex of this square?
A) $(2; 0)$ **B)** $(2; 3)$ **C)** $(2; -6)$ **D)** $(3; 5)$ **E)** $(3; -1)$
9. Which of the following polynomials is not divisible by polynomial $x + 1$?
A) $x^2 + 3x + 2$ **B)** $x^4 - x^2$ **C)** $2x^2 + 3x + 1$ **D)** $-x^2 + 3x + 4$ **E)** $x^3 + x$
10. In the year number 2014 the digits are different and the last digit is greater than the sum of the other three digits. How many years ago did this occur the last time?
A) 5 **B)** 215 **C)** 305 **D)** 395 **E)** 485



17. The vertices of a cube are numbered 1 to 8 in such a way that the result of adding the four numbers of the vertices of a face is the same for all faces. Numbers 1, 4 and 6 are already set on some vertices as shown. What is the value of x ?
A) 2 **B)** 3 **C)** 5 **D)** 7 **E)** 8
18. The label on a package of cream cheese reads: 24 % total fat. The same label also reads: 64 % fat in dry matter. What is the percentage of water in this cheese?
A) 88% **B)** 62.5% **C)** 49% **D)** 42% **E)** 37.5%
19. Line l passes through the vertex A of a rectangle $ABCD$. The distance from point C to l is 2, and the distance from point D to l is 6. If AD is twice AB , find AD .

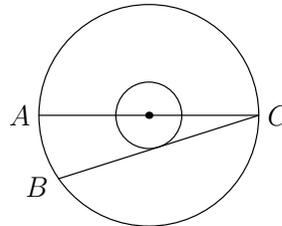


Questions for 4 points

11. The size of a rectangular box is $a \times b \times c$, with $a < b < c$. If you increase a or b or c by a given positive number d , the volume of the box also increases. In which of the following cases is the increase of the volume of the box the greatest?
A) If you increase a **B)** If you increase b **C)** If you increase c
D) The increase of the volume is the same in **A**, **B**, **C**
E) It depends on the values of a , b , c , d
12. In a football match, the winner gets 3 points, the loser gets 0 points, while in the case of a draw, each team gets 1 point. Four teams, A , B , C , D take part in a football tournament. Each team plays three games: one against each other team. At the end of the tournament team A has 7 points and teams B and C have 4 points each. How many points does team D have?
A) 0 **B)** 1 **C)** 2 **D)** 3 **E)** 4

- A)** 10 **B)** 12 **C)** 14 **D)** 16 **E)** $4\sqrt{3}$

13. The radii of two concentric circles are in proportion 1 : 3. AC is a diameter of the big circle; BC is a chord of the big circle which is tangent to the smaller; and the length of AB is 12. Then the radius of the big circle is
A) 13 **B)** 18 **C)** 21 **D)** 24 **E)** 26



14. How many triples (a, b, c) of integers with $a > b > c > 1$ satisfy $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} > 1$?
A) 0 **B)** 1 **C)** 2 **D)** 3 **E)** Infinitely many

15. a , b , c are nonzero numbers and n is a positive integer. It is known that the numbers $(-2)^{2n+3}a^{2n+2}b^{2n-1}c^{3n+2}$ and $(-3)^{2n+2}a^{4n+1}b^{2n+5}c^{3n-4}$ have the same sign. Which of the following is definitely true?
A) $a > 0$ **B)** $b > 0$ **C)** $c > 0$ **D)** $a < 0$ **E)** $b < 0$

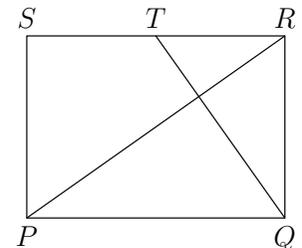
16. Six weeks is $n!$ seconds. Find n .
A) 6 **B)** 7 **C)** 8 **D)** 10 **E)** 12

20. The function $f(x) = ax + b$ satisfies the equalities $f(f(f(1))) = 29$ and $f(f(f(0))) = 2$. What is the value of a ?
A) 1 **B)** 2 **C)** 3 **D)** 4 **E)** 5

Questions for 5 points

21. There are 10 different positive integers, exactly 5 of them are divisible by 5 and exactly 7 of them are divisible by 7. Let M be the greatest of these 10 numbers. What is the minimum possible value of M ?
A) 105 **B)** 77 **C)** 75 **D)** 63 **E)** None of these

22. $PQRS$ is a rectangle. T is the midpoint of RS . QT is perpendicular to the diagonal PR . What is the ratio $PQ : QR$?
A) 2 : 1 **B)** $\sqrt{3} : 1$ **C)** 3 : 2 **D)** $\sqrt{2} : 1$ **E)** 5 : 4



23. There are 9 kangaroos called Greatkangs. They are coloured either silver or gold. When 3 Greatkangs meet by chance, there is a two in three chance that none of them is silver. How many Greatkangs are gold?
A) 1 **B)** 3 **C)** 5 **D)** 6 **E)** 8