

KANGAROO 2009



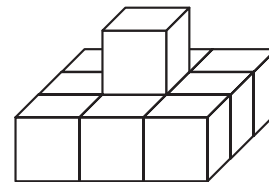
Nipper
1 and 2 grades

Time allowed: 50 min

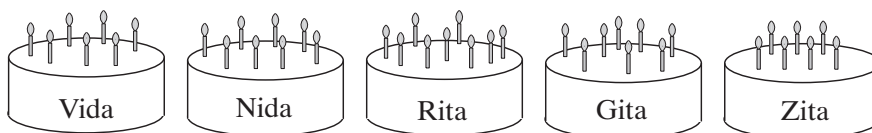
Calculators are not permitted

3-point questions

1. The following construction is made from the identical wooden tiles (as shown in the picture). From how many tiles?
- A) 12 B) 8 C) 9 D) 10 E) 11



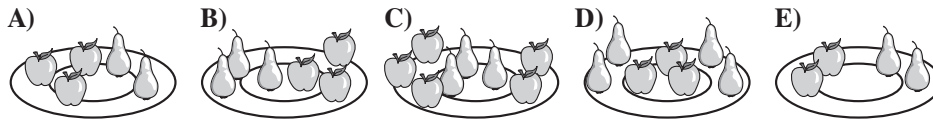
2. What is the sum of all digits of the number 2009?
- A) 7 B) 11 C) 12 D) 18 E) 209
3. 5 girls have their birthday on the same day. Their birthday cakes are shown.




Which of the girls is the eldest one?

- A) Vida B) Nida C) Rita D) Gita E) Zita

4. On which plate are there less apples than pears?

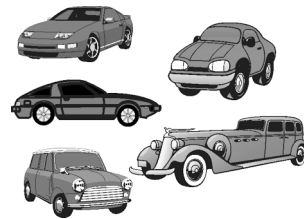


5. In the table Ann has written 4 numbers, the sum of which is 50. Which number is covered by the butterfly?

| | |
|----|---|
| 5 |  |
| 20 | 17 |

A) 18 B) 3 C) 9 D) 13 E) 8

6. Peter has 12 toycars and Paul has 4 toycars more. How many toycars have the boys got together?



A) 28 B) 16 C) 48 D) 20 E) 8

4-point questions

7. On the last school day the father with 3 sons went to the circus.

| Ticket Office | |
|---------------|-------|
| Child ticket | 9 Lt |
| Adult ticket | 12 Lt |

How much litas did the father pay for all the tickets?

A) 48 B) 21 C) 39 D) 30 E) Another answer

8. Ann wrote down two arithmetical operations correctly. Some of the numbers she has hidden under stickers – identical numbers under the identical stickers:

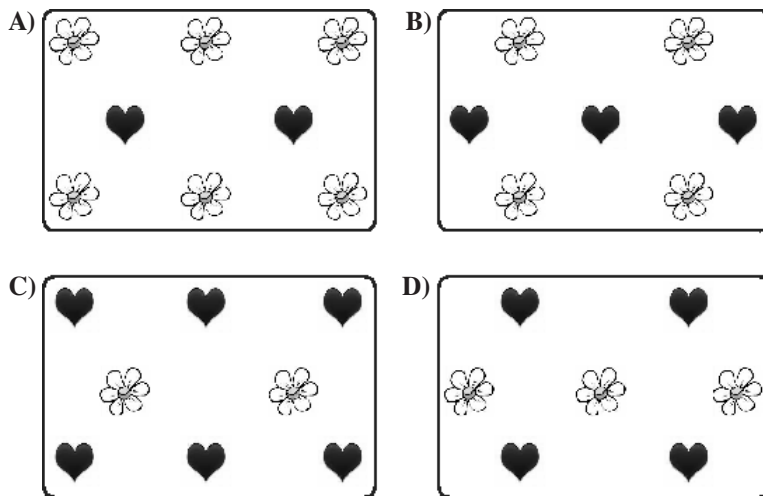
$$21 - 7 = \text{☀}$$

$$2 \cdot \text{☀} = \text{☀} + 1$$

What number is covered under the sticker ☀?

A) 15 B) 14 C) 25 D) 27 E) 28

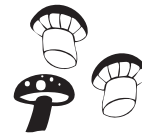
9. The doctor prescribed 60 tablets for Ann to be taken one tablet each day. Ann took the first tablet on Monday. On which day of the week will Ann take the last tablet?
 A) On Monday B) On Tuesday C) On Wednesday D) On Thursday
 E) On Friday
10. Diana's mother bought 6 identical packets of chalks. Diana spilled the content of 2 packets – there were 18 chalks on the floor. How many chalks did Diana's mother buy?
 A) 26 B) 54 C) 24 D) 108 E) 9
11. Tom is 2 cm taller than Peter and 5 cm taller than Paul. How many centimeters is Peter taller than Paul?
 A) 7 cm B) 3 cm C) 10 cm D) Paul is higher than Peter
 E) Impossible to determine
12. Diana has drawn 6 flowers and Ann has drawn 4 hearts. Barbara has drawn 3 times less flowers than Diana and 2 hearts more than Ann. Which of the pictures below is drawn by Barbara?



E) None

5-point questions

13. There are 19 monkeys in the zoo: 4 of them are chimpanzees and 3 of them are baboons. All the rest monkeys, capuchins, are evenly distributed between three cages. How many capuchins are there in each cage?
A) 5 B) 7 C) 3 D) 6 E) 4
14. Johnny is 4 years old, and his father is 26 years old. How old will Johnny's father be when Johnny grows thrice older than now?
A) 78 B) 38 C) 42 D) 34 E) Another answer
15. Granny baked cakes with cheese and cakes with jam – 31 cake in total. If there were 11 additional cakes with cheese, the number of cakes with cheese and that with jam would be the same. How many cakes with cheese has Granny baked?
A) 10 B) 21 C) 20 D) 15 E) Another answer
16. Ann has bought 2 identical notebooks and got the change 4 litas. If she bought 2 notebooks more she would be short of 2 litas. How much does one notebook cost?
A) 2 litas B) 10 litas C) 6 litas D) 3 litas E) Another answer
17. Adam, Michael, Paul, and Tom are showing their post-stamps. It appears that Michael has more stamps than Paul, and Tom has less than Adam. It is known that it is not Tom who has the least amount of stamps. Which of the boys has the least number of stamps?
A) Adam B) Michael C) Paul D) Tom E) Impossible to determine
18. Dad has been mushrooming for two hours. During the first hour he picked up 39 mushrooms. How many mushrooms did he pick up during the second hour, if mummy, cleaning 7 mushrooms in 5 minutes, has prepared all the picked up mushrooms in 40 minutes?



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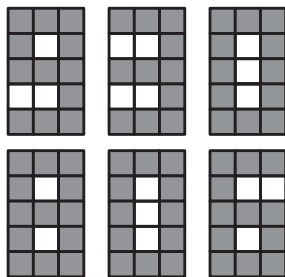
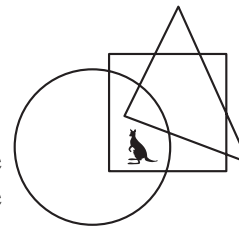


Minor
3 and 4 grades

Time allowed: 75 min
Calculators are not permitted

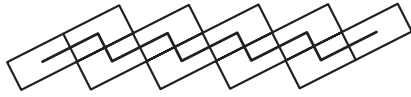
3-point questions

- $200 \cdot 9 + 200 + 9 =$
A) 418 B) 1909 C) 2009 D) 4018 E) 20009
- Where is the kangaroo?
A) In the circle and in the triangle, but not in the square
B) In the circle and in the square, but not in the triangle
C) In the triangle and in the square, but not in the circle
D) In the circle, but neither in the square nor in the triangle
E) In the square, but neither in the circle nor in the triangle
- There are five brothers in a family and each of them has one sister. How many brothers and sisters together are there in this family?
A) 6 B) 7 C) 8 D) 9 E) 10
- There is a number 930 on the display (see the picture). How many little square lights must be switched in order to obtain number 806?



- A) 5 B) 6 C) 7 D) 8 E) 9
- Mom has bought 16 mandarins. Karol ate half of them, Eva ate two and Dana ate the rest. How many mandarins has Dana eaten?
A) 4 B) 6 C) 8 D) 10 E) 12

6. In his garden Anthony has made a path shown in the figure, using 10 tiles of size $4\text{ dm} \times 6\text{ dm}$. Anthony has drawn a bold line between the midpoints of the tiles.



How long is the bold line?

- A) 25 dm B) 40 dm C) 46 dm D) 50 dm E) 92 dm
7. Sofiko threw a dice four times and she obtained a total of 23 points. How many times did she get 6 points?
A) 0 B) 1 C) 2 D) 3 E) 4
8. A movie is 90 minutes long. It started at 17:10. In the middle there were two commercial breaks, one lasting eight minutes and the other five minutes. At what time did the movie finish?
A) 18:13 B) 18:27 C) 18:47 D) 18:53 E) 19:13

4-point questions

9. There are 25 boys and 19 girls in the dance group. Every week 2 more boys and 3 more girls join the dance group. After how many weeks will there be the same number of boys and girls in the dance group?
A) 6 B) 5 C) 4 D) 3 E) 2
10. Peter was dividing a chocolate. He broke one row of 5 pieces for his brother and then one row of 7 pieces for his sister in a way you see on the picture. How many pieces did the whole bar of chocolate consist of?
A) 28 B) 32 C) 35 D) 40 E) 54



11. A white pig and a black one weigh 320 kilos altogether. The black pig weighs 32 kilos more than the white pig. How much does the white pig weigh?
A) 104 kg B) 87 kg C) 52 kg D) 96 kg E) 53 kg
12. Nine numbers are written in the cells of 3×3 table (see the figure). Per move any two numbers can be interchanged. What is the smallest number of such moves to obtain the table for which the sum of the numbers in any row is divisible by 3?
A) 1 B) 2 C) 3 D) 4 E) It is impossible to obtain such table

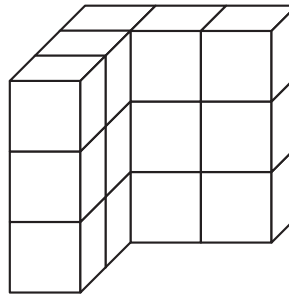
| | | |
|---|----|---|
| 4 | 5 | 1 |
| 8 | 10 | 4 |
| 7 | 1 | 2 |

13. One side of the rectangle is 8 cm long, while the other is half as long. How long is a side of the square, the perimeter of which is the same as that of the rectangle?

- A) 4 cm B) 6 cm C) 8 cm D) 12 cm E) 24 cm

14. Thomas made a wall from small cubes (see the picture). How many cubes did he use?

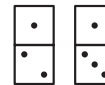
- A) 6 B) 12 C) 13 D) 15 E) 16



15. Three squirrels Anni, Asia, and Elli collected 7 nuts. They all collected a different number of nuts, but each of them found at least one. Anni collected the least number of nuts and Asia most of all. How many nuts did Elli find?

- A) 1 B) 2 C) 3 D) 4 E) It is impossible to determine

16. Which figure cannot be formed from the two dominoes illustrated on the right?



- A) B) C) D) E)

5-point questions

17. A farmer has 30 cows, some chickens, but no other animals. The total number of legs of the chickens is equal to the total number of legs of the cows. How many animals altogether does the farmer have?

- A) 60 B) 90 C) 120 D) 180 E) 240

18. Ann and Peter live on the same street. On one side of Ann's house there are 27 houses and on the other side there are 13 houses. Peter lives in the house that is exactly in the middle of the street. How many houses are in between Ann's and Peter's house?

- A) 6 B) 7 C) 8 D) 14 E) 21

19. A secret agent wants to guess a 6-digit code. He knows that the sum of the digits in the even positions is equal to the sum of the digits in the odd positions. Which of the following numbers could be the code?

- A) 81**61 B) 7*727* C) 4*4141 D) 12*9*8 E) 181*2*

20. Meta collects photos of famous sportsmen. Each year the number of her photos equals the sum of the numbers of her photos of the previous two years. In 2008 she had 60 photos and this year she has 96 photos. How many photos did she have in 2006?

- A) 20 B) 24 C) 36 D) 40 E) 48

21. There are 1 red, 1 blue, 1 yellow, and 1 white flower in the garden. Maja the bee visits each flower only once. She starts from the red flower, but she does not fly directly from the yellow one to the white one. In how many ways can Maja visit all the flowers?

- A) 1 B) 2 C) 3 D) 4 E) 5

22. At 6:15 a ghost vanished, and the mad clock that showed the right time so far, started running at the right speed, but backwards. The ghost appeared again at 19:30. What time did the mad clock show at the moment the ghost has appeared again?

- A) 17:00 B) 17:45 C) 18:30 D) 19:00 E) 19:15

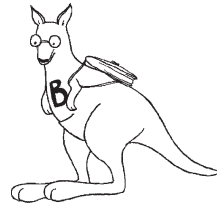
23. Sylvia draws figures consisting of lines which are each 1 cm long. At the end of each line she always turns at 90° either to the left or to the right. At each turn to the right she draws a symbol ♡ and at each turn to the left she draws ♠ on a separate sheet of paper. One day she drew a figure and the following sequence of symbols: ♡ ♠ ♠ ♠ ♡ ♡. Which of the following figures could she draw?



24. In the land Funnyfeet, the left foot of each man is one or two sizes bigger than his right foot. Nevertheless, the shoes are always sold in pairs of the same size. To save money, a group of friends bought shoes together. After all of them have put on the shoes that fitted them, there were exactly two shoes left, one of size 36 and the another of size 45. What is the smallest possible number of people in the group?

- A) 5 B) 6 C) 7 D) 8 E) 9

KANGAROO 2009



Benjamin
5 and 6 grades

Time allowed: 75 min

Calculators are not permitted

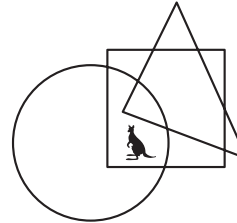
3-point questions

1. Among these numbers, which is even?

- A) 2009 B) $2 + 0 + 0 + 9$ C) $200 - 9$ D) 200×9 E) $200 + 9$

2. Where is the kangaroo?

- A) In the circle and in the triangle, but not in the square
B) In the circle and in the square, but not in the triangle
C) In the triangle and in the square, but not in the circle
D) In the circle, but neither in the square nor in the triangle
E) In the square, but neither in the circle nor in the triangle



3. How many integers are there between 2.008 and 20.09?

- A) 17 B) 18 C) 19 D) 16 E) More than 19

4. The smallest number of digits to be erased in the number 12323314 in order to get a number that reads identically from left to right and from right to left, is equal to

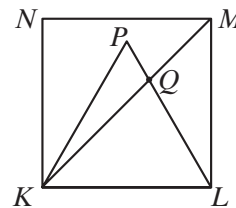
- A) 1 B) 2 C) 3 D) 4 E) 5

5. There are three boxes: white, red and green. One of them contains a bar of chocolate, the second contains an apple, and the third is empty. Find the chocolate, if it is known, that the chocolate is either in the white or in the red box, and the apple is neither in the white nor in the green box.

- A) White B) Red C) Green D) Red or green E) Impossible to determine

6. $KLMN$ is a square and KLP is an equilateral triangle. What is the measure of $\angle LQM$?

- A) 95° B) 105° C) 115° D) 125° E) 135°

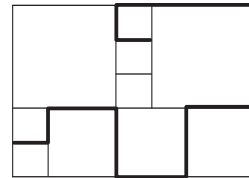


7. A bridge is built across the river. The river is 120 meters wide. One quarter of the bridge is over the left river bank and another quarter of the bridge is over the right river bank. How long is the bridge?

- A) 150 m B) 180 m C) 210 m D) 240 m E) 270 m

8. There are squares of three different sizes in the picture. The side of the smallest one is 20 cm long. How long is the marked broken line?

A) 380 cm B) 400 cm C) 420 cm D) 440 cm E) 1680 cm

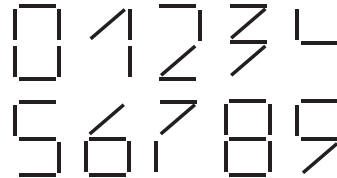


9. There are cats and dogs in the room. The number of cats' paws is twice the number of dogs' noses. Then the number of cats is

A) twice the number of dogs B) equal to the number of dogs C) half the number of dogs D) $\frac{1}{4}$ the number of dogs E) four times the number of dogs

10. We use identical small sticks to form digits, as shown on the right. Given a number, by the *weight* of it we mean the number of sticks needed to compose it. What is the weight of the heaviest 2-digit number?

A) 10 B) 11 C) 12 D) 13 E) 14



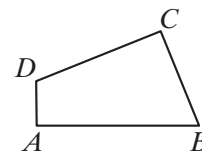
4-point questions

11. How many positive integers n have the property that $n + 2$ is a divisor of number 78?

A) 8 B) 7 C) 6 D) 5 E) 4

12. The quadrilateral $ABCD$ has sides $AB = 11$, $BC = 7$, $CD = 9$ and $DA = 3$ and right angles in A and C . What is the area of this quadrilateral?

A) 30 B) 44 C) 48 D) 52 E) 60

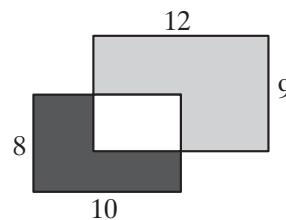


13. There are 39 boys and 23 girls in the dance group. Every week 6 more boys and 8 more girls join the dance group. After a few weeks there will be the same number of boys and girls in the dance group. How many boys and girls all in all will be then in the dance group?

A) 144 B) 154 C) 164 D) 174 E) 184

14. Two rectangles of 8×10 and 9×12 partly cover each other. The dark area is 37. What is the grey area?

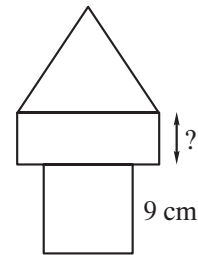
A) 60 B) 62 C) 62.5 D) 64 E) 65



15. Eight cards numbered from 1 to 8 are put in the boxes M and N so that both sums of the card numbers in each box are equal. If there are only 3 cards in the box M, then you can be sure that

A) three cards in box N are odd numbered
 B) four cards in box N are even numbered
 C) card number 1 is not in box N
 D) card number 2 is in box N
 E) card number 5 is in box N

16. In the picture a “tower” is formed of three structures – square, rectangle, and equilateral triangle. The perimeter of all the three structures is the same. The side of the square is 9 cm long. How long is marked side of the rectangle?



A) 4 cm B) 5 cm C) 6 cm D) 7 cm E) 8 cm

17. We want to fill up a $40 \times 40 \times 60$ box with rigid cubes all of the same size. Which is the minimum number of cubes that allows us to do that?
 A) 96 B) 96 000 C) 12 D) 12 000 E) 768
18. Today is Sunday. Francis begins reading a book of 290 pages. He reads 4 pages each day, except Sundays, on which he always reads 25 pages. How many days will it take him to read the book?
 A) 15 B) 46 C) 40 D) 35 E) 41
19. Andrija, Branimir, Celestin and Davor have won the first four places at the fencing tournament. If you add the number of places won by Andrija, Branimir and Davor, you will get number 6. You will get the same number if you add the number of places won by Branimir and Celestin. Who won the first place, if Branimir is ranked higher than Andrija?
 A) Andrija B) Branimir C) Celestin D) Davor E) Impossible to determine
20. Oliver takes 2009 equally sized square pieces and places them all side by side in the form of a full rectangle. How many different rectangles can he have?
 A) 1 B) 2 C) 3 D) 5 E) 10

5-point questions

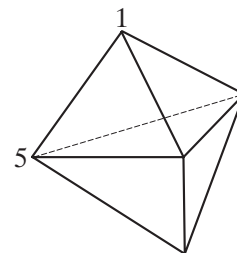
21. There are 4 statements about the positive integer M :

M is divisible by 5; M is divisible by 11;
 M is divisible by 55; M is less than 10.

It is known that two of these statements are true, and the other two are false. Then M can be equal to:

A) 0 B) 5 C) 10 D) $11 \cdot 55$ E) 55

22. The picture shows a solid formed with 6 triangular faces. At each vertex there is a number. For each face we consider the sum of the 3 numbers at the vertices of that face. If all the sums are the same and two of the numbers are 1 and 5, as shown, what is the sum of all the 5 numbers?



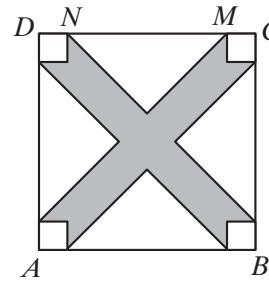
A) 9 B) 12 C) 17 D) 18 E) 24

23. The rooms of a hotel are numbered with three digits. The first indicates the floor and the next two the number of the room. For example, 125 indicates room 25 of the first floor. If the hotel has a total of 5 floors numbered from 1 to 5 with 35 rooms per floor, how many times will the digit 2 be used to number all the rooms?

A) 60 B) 65 C) 95 D) 100 E) 105

24. $ABCD$ is a square with a side 10 cm long. The distance from point N to point M is 6 cm. Four of the non-shaded regions are equal isosceles triangles and other four are equal squares. Find the area of the shaded region.

A) 42 cm^2 B) 46 cm^2 C) 48 cm^2 D) 52 cm^2 E) 58 cm^2



25. The total of each row and column is given. What is the value of $a + b - c$?

A) 4 B) 5 C) 6 D) 7 E) 8

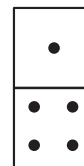
| | | | |
|-----|-----|-----|----|
| a | b | a | 11 |
| b | a | c | 8 |
| b | c | a | 8 |
| 10 | 8 | 9 | |

26. Jadwiga has multiplied 18 multipliers equal 8 and 50 multipliers equal 5. How many digits has the product?

A) 13 B) 40 C) 52 D) 60 E) 100

27. A complete set of 28 dominoes contains every possible combination of two numbers of pips between 0 and 6 included, including twice the same number. How many pips are there altogether on the set of dominoes?

A) 84 B) 105 C) 126 D) 147 E) 168



28. In a 4×2 table, two numbers are written in the first row. Each next row contains the sum and the difference of the numbers written in the previous row (see the picture for an example). In a table 7×2 , filled in the same way, the numbers of the last row are 94 and 64. What is the sum of the numbers in the first row?

A) 8 B) 10 C) 12 D) 20 E) 24

| | |
|----|----|
| 10 | 3 |
| 13 | 7 |
| 20 | 6 |
| 26 | 14 |

29. In the land Funnyfeet, the left foot of each man is one or two sizes bigger than his right foot. Nevertheless, the shoes are always sold in pairs of the same size. To save money, a group of friends bought shoes together. After all of them have put on the shoes that fitted them, there were exactly two shoes left, one of size 36 and the another of size 45. What is the smallest possible number of people in the group?

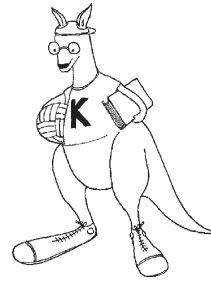
A) 5 B) 6 C) 7 D) 8 E) 9

30. We want to color the squares in the grid using colors a , b , c and d so that the neighboring squares were not of the same color (squares that share a vertex are considered neighbors). Some of the squares have been colored as shown. What are the possibilities for the shaded square?

| | | | | |
|-----|-----|--|-----|-----|
| a | b | | c | d |
| | | | | |
| | | | | |
| | | | | |

- A) Only a B) Only b C) Only c D) Only d E) There are two different possibilities

KANGAROO 2009



Cadet
7 and 8 grades

Time allowed: 75 min

Calculators are not permitted

3-point questions

1. Which of these numbers is even?

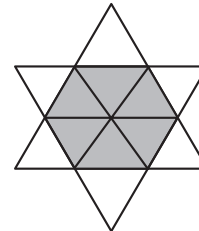
- A) 2009 B) $2 + 0 + 0 + 9$ C) $200 - 9$ D) 200×9 E) $200 + 9$

2. There were 4 boys and 4 girls at a party. The boys danced only with girls and the girls danced only with boys. Afterwards we asked all of them, how many dance partners each of them had. The boys said: 3, 1, 2, 2. Three of the girls said: 2, 2, 2. What number did the fourth girl say?

- A) 0 B) 1 C) 2 D) 3 E) 4

3. The star in the picture is formed from 12 identical small equilateral triangles. The perimeter of the star is 36 cm. What is the perimeter of the shaded hexagon?

- A) 6 cm B) 12 cm C) 18 cm D) 24 cm E) 30 cm

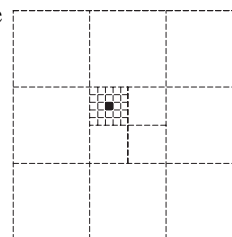


4. Harry delivers folders in the Long Street. He must deliver a folder to all the houses with an odd number. The first house has number 15, the last one has number 53. How many houses does Harry visit?

- A) 19 B) 20 C) 27 D) 38 E) 53

5. The area of the big square is 1. What is the area of the black little square?

- A) $\frac{1}{100}$ B) $\frac{1}{300}$ C) $\frac{1}{600}$ D) $\frac{1}{900}$ E) $\frac{1}{1000}$



6. The product of four different positive integers is 100. What is their sum?

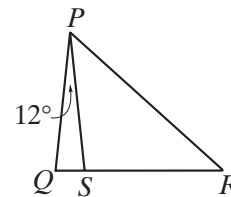
- A) 10 B) 12 C) 15 D) 18 E) 20

7. There are cats and dogs in the room. The number of cats' paws is twice the number of dogs' noses. Then the number of cats is

- A) twice the number of dogs B) equal to the number of dogs C) half the number of dogs
 D) $\frac{1}{4}$ the number of dogs E) four times the number of dogs

8. In the figure on the right, QSR is a straight line, $\angle QPS = 12^\circ$ and $PQ = PS = RS$. What is the size of $\angle QPR$?

- A) 36° B) 42° C) 54° D) 60° E) 84°



9. The elevator can take either 12 adults or 20 children. How many children at most could go up with 9 adults?

- A) 3 B) 4 C) 5 D) 6 E) 8

10. At 6:15 a ghost vanished, and the mad clock that showed the right time so far, started running at the right speed, but backwards. The ghost appeared again at 19:30. What time did the mad clock show at the moment the ghost has appeared again?

- A) 17:00 B) 17:45 C) 18:30 D) 19:00 E) 19:15

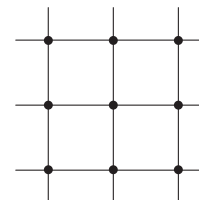
4-point questions

11. How many positive integers have as many digits in the decimal representation of their square as of their cube?

- A) 0 B) 3 C) 4 D) 9 E) Infinitely many

12. What is the smallest number of bold points in the figure one needs to remove so that no 3 of the remaining points were collinear?

- A) 1 B) 2 C) 3 D) 4 E) 7

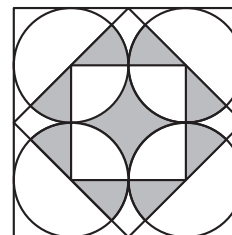


13. Nick measured all the 6 angles in two triangles – one acute-angled and one obtuse-angled. He remembered four of those angles: 120° , 80° , 55° , and 10° . What is the smallest angle of the acute-angled triangle?

- A) 5° B) 10° C) 45° D) 55° E) Impossible to determine

14. Which part of the outer square is shaded?

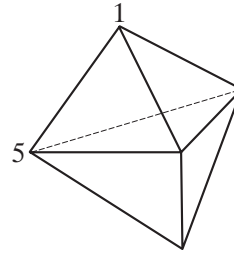
- A) $\frac{1}{4}$ B) $\frac{\pi}{12}$ C) $\frac{\pi + 2}{16}$ D) $\frac{\pi}{4}$ E) $\frac{1}{3}$



15. 25 people are standing in a queue on the island of nobles and liars. Everyone, except the first person in the queue, said that the person ahead of him in the queue was a liar, and the first man in the queue said that all the people standing behind him were liars. How many liars were there in the queue? (Nobles always speak the truth, and liars always tell lies.)

- A) 0 B) 12 C) 13 D) 24 E) Impossible to determine

16. The picture shows a solid formed with 6 triangular faces. At each vertex there is a number. For each face we consider the sum of the 3 numbers at the vertices of that face. If all the sums are the same and two of the numbers are 1 and 5, as shown, what is the sum of all the 5 numbers?
- A) 9 B) 12 C) 17 D) 18 E) 24

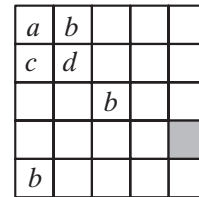


17. In the equality

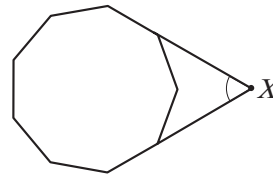
$$\frac{E \cdot I \cdot G \cdot H \cdot T}{F \cdot O \cdot U \cdot R} = T \cdot W \cdot O$$

different letters stand for different digits while the same letters stand for the same digits. How many different values can the product $T \cdot H \cdot R \cdot E \cdot E$ have?

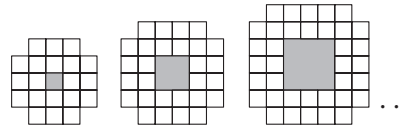
- A) 1 B) 2 C) 3 D) 4 E) 5
18. We want to color the squares in the grid using colors a , b , c , and d so that neighboring squares were not of the same color (squares that share a vertex are considered neighbors). Some of the squares have been colored as shown. What are the possibilities for the shaded square?
- A) Only a or b B) Only c C) Only d
 D) Only c or d E) Any of a , b , c , d



19. The diagram shows a regular enneagon (9-sided polygon). What is the size of the marked angle at X ?
- A) 40° B) 45° C) 50° D) 55° E) 60°



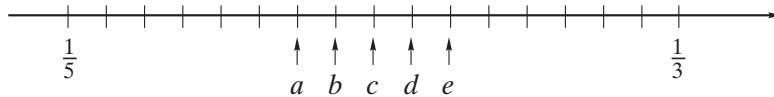
20. The first three patterns are shown. How many white unit squares are needed to build the 10th pattern in this sequence?
- A) 76 B) 80 C) 84 D) 92 E) 100



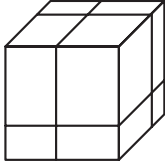
5-point questions

21. There are 4 statements about the positive integer M :
 M is divisible by 5; M is divisible by 11;
 M is divisible by 55; M is less than 10.
 It is known that two of these statements are true, and the other two are false. Then M can be equal to:
 A) 0 B) 5 C) 10 D) 11 · 55 E) 55
22. How many ten-digit numbers, composed only of digits 1, 2, or 3, do there exist, in which any two neighboring digits differ by 1?
 A) 16 B) 32 C) 64 D) 80 E) 100

23. The fractions $\frac{1}{3}$ and $\frac{1}{5}$ are placed on a number-line.



Where is the fraction $\frac{1}{4}$?

- A) *a* B) *b* C) *c* D) *d* E) *e*
24. Three cuts are made through a large cube to get eight smaller cuboids. What is the ratio of the total surface area of these eight cuboids to the surface area of the original cube?
- A) 1:1 B) 4:3 C) 3:2 D) 2:1 E) 4:1
- 
25. All the divisors of number N , unequal to N and to 1, were written in turn. It occurred that the greatest of the divisors in the line is 45 times as great as the smallest one. How many numbers N satisfy this condition?
- A) 0 B) 1 C) 2 D) More than 2 E) Impossible to determine
26. A square has been dissected into 2009 squares whose lengths of the sides are integers. What is the shortest possible length of the side of the original square?
- A) 44 B) 45 C) 46 D) 503
E) It is not possible to dissect a square into 2009 squares of this type
27. In the quadrilateral $PQRS$, $PQ = 2006$, $QR = 2008$, $RS = 2007$ and $SP = 2009$. Which interior angles of the quadrilateral are necessarily smaller than 180° ?
- A) P, Q, R B) Q, R, S C) P, Q, S D) P, R, S E) P, Q, R, S
28. If I place a $6\text{ cm} \times 6\text{ cm}$ square on a triangle, I can cover up to 60% of the triangle. If I place the triangle on the square, I can cover up to $\frac{2}{3}$ of the square. What is the area of the triangle?
- A) $22\frac{4}{5}\text{ cm}^2$ B) 24 cm^2 C) 36 cm^2 D) 40 cm^2 E) 60 cm^2
29. Man Friday wrote down in a row several different integers smaller than 11. Robinson Crusoe examined these numbers and noticed with satisfaction that in each pair of the neighbouring numbers one of the numbers was divisible by another. How many numbers at most could Man Friday write down?
- A) 6 B) 7 C) 8 D) 9 E) 10
30. In a triangle ABC , the angle B is 20° and the angle C is 40° . The length of the bisector of the angle A is 2. Find $BC - AB$.
- A) 1 B) 1.5 C) 2 D) 4 E) Impossible to determine

KANGAROO 2009



Junior
9 and 10 grades

Time allowed: 75 min

Calculators are not permitted

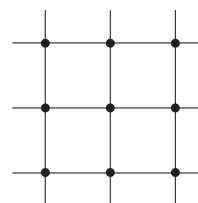
3-point questions

1. Which of these numbers is a multiple of 3?

- A) 2009 B) $2 + 0 + 0 + 9$ C) $(2 + 0) \cdot (0 + 9)$ D) 2^9 E) $200 - 9$

2. What is the smallest number of bold points in the figure one needs to remove so that no 3 of the remaining points were collinear?

- A) 1 B) 2 C) 3 D) 4 E) 7



3. 2009 people participated in a popular race. The number of people John has outstripped was three times larger than the number of people who won over John. In what place has John been ranked in the race?

- A) 503 B) 501 C) 500 D) 1503 E) 1507

4. What is the value of the $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$ of 1000?

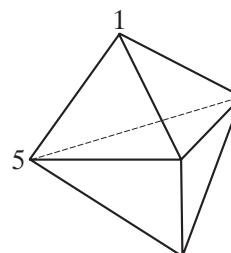
- A) 250 B) 200 C) 100 D) 50 E) None of these

5. A long sequence of digits has been composed by writing the number 2009 repeatedly 2009 times. The sum of those odd digits in the sequence that are immediately followed by an even digit is equal to

- A) 2 B) 9 C) 4018 D) 18072 E) 18081

6. The picture shows a solid formed with 6 triangular faces. At each vertex there is a number. For each face we consider the sum of the 3 numbers at the vertices of that face. If all the sums are the same and two of the numbers are 1 and 5, as shown, what is the sum of all the 5 numbers?

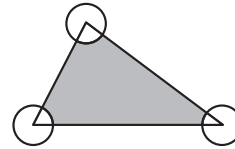
- A) 9 B) 12 C) 17 D) 18 E) 24



7. How many positive integers have as many digits in the decimal representation of their square as of their cube?

- A) 0 B) 3 C) 4 D) 9 E) Infinitely many

8. The area of the triangle in the picture is 80 m^2 and the radius of the circles centered at the vertices is 2 m. What is the measure, in m^2 , of the shaded area?



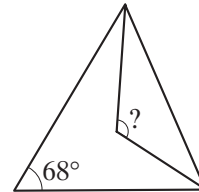
- A) 76 B) $80 - 2\pi$ C) $40 - 4\pi$ D) $80 - \pi$ E) 78π

9. Leonard has written a sequence of numbers so that each number (from the third number in the sequence) was a sum of the previous two numbers in the sequence. The fourth number in the sequence was 6 and the sixth number in the sequence was 15. What was the seventh number in the sequence?

- A) 9 B) 16 C) 21 D) 22 E) 24

10. A triangle has an angle of 68° . Bisectors of the other two angles are drawn. How many degrees is the angle with the question sign?

- A) 120° B) 124° C) 128° D) 132° E) 136°



4-point questions

11. At each test, the mark can be 0, 1, 2, 3, 4, or 5. After 4 tests, Mary's average was 4. One of the sentences cannot be true. Which is it?

- A) Mary got all the marks 4
 B) Mary got the mark 3 exactly twice
 C) Mary got the mark 3 exactly 3 times
 D) Mary got the mark 1 exactly once
 E) Mary got the mark 4 exactly twice

12. The Borromean rings have a surprising property: the three of them cannot be separated without destroying them, but once one of them is removed (regardless which one), the other two are not linked anymore. Which of the following figures shows the Borromean rings?



- A) A B) B C) C D) D E) E

13. 25 people are standing in a queue on the island of nobles and liars. Everyone, except the first person in the queue, said that the person ahead of him in the queue was a liar, and the first man in the queue said that all the people standing behind him were liars. How many liars were there in the queue? (Nobles always speak the truth, and liars always tell lies.)

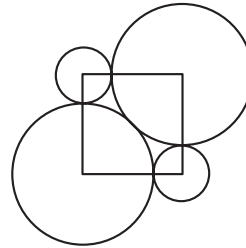
- A) 0 B) 12 C) 13 D) 24 E) Impossible to determine

14. If $a \square b = ab + a + b$, and $3 \square 5 = 2 \square x$, then x equal is to:

- A) 3 B) 6 C) 7 D) 10 E) 12

15. Around the vertices of a square circles are drawn: 2 large and 2 small ones. The large circles are tangent to each other and to both the small circles. What is the ratio between the radius of a large circle and that of a small circle?

A) $\frac{2}{9}$ B) $\sqrt{5}$ C) $1 + \sqrt{2}$ D) 2.5 E) 0.8π



16. The difference between \sqrt{n} and 10 is less than 1. How many such integer n exist?

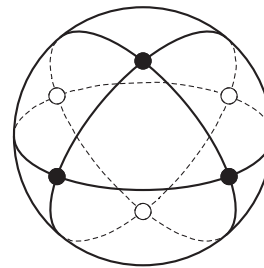
A) 19 B) 20 C) 39 D) 40 E) 41

17. Man Friday wrote down in a row several different natural numbers smaller than 11. Robinson Crusoe examined these numbers and noticed with satisfaction that in each pair of the neighboring numbers one of the numbers was divisible by another. How many numbers at most could Man Friday write down?

A) 6 B) 7 C) 8 D) 9 E) 10

18. 3 circular hoops are joined together so that they intersect at the right angles as shown. A ladybird lands on an intersection and crawls as follows: it travels along a quarter-circle, turns to the right 90° , then travels along another quarter-circle and turns to the left 90° . Proceeding in this way, how many quarter-circles will it travel along before she returns again to her starting point?

A) 6 B) 9 C) 12 D) 15 E) 18



19. How many zeros should be inscribed instead of * in the decimal fraction $1.*1$ in order to get a number that is smaller than $\frac{2009}{2008}$, but larger than $\frac{20009}{20008}$?

A) 1 B) 2 C) 3 D) 4 E) 5

20. If $a = 2^{25}$, $b = 8^8$ and $c = 3^{11}$, then

A) $a < b < c$ B) $b < a < c$ C) $c < b < a$ D) $c < a < b$ E) $b < c < a$

5-point questions

21. How many ten-digit numbers, composed only of digits 1, 2 and 3, do there exist, in which any two neighboring digits differ by 1?

A) 16 B) 32 C) 64 D) 80 E) 100

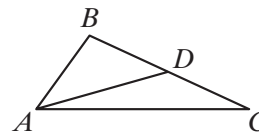
22. A young kangaroo has 2009 unit $1 \times 1 \times 1$ cubes that he has used in forming a cuboid. He also has 2009 stickers 1×1 that he must use to color the outer surface of the cuboid. The kangaroo has achieved his goal and some stickers were left. How many stickers were left?

A) More than 1000 B) 763 C) 476 D) 49 E) 0

23. Bob wants to place draughts into cells of the 4×4 board so that the numbers of the draughts in any row and any column were different (more than one draught can be placed into one cell as well as the cell can be empty). What is the smallest possible number of the draughts placed on the board?

A) 13 B) 14 C) 15 D) 16 E) 20

24. Some oranges, peaches, apples, and bananas were put in a row so that somewhere in the row each type of fruit can be found side by side with each other type of fruit. What is the least number of fruits in the row?
 A) 4 B) 5 C) 8 D) 11 E) 12
25. What is the least integer n , for which $(2^2 - 1) \cdot (3^2 - 1) \cdot (4^2 - 1) \cdot \dots \cdot (n^2 - 1)$ is a perfect square?
 A) 6 B) 8 C) 16 D) 27 E) Another answer
26. All the divisors of the number N , unequal to N and to 1, were written in turn. It occurred that the greatest of the divisors in the line is 45 times greater than the smallest one. How many numbers satisfy this condition?
 A) 0 B) 1 C) 2 D) More than 2 E) Impossible to determine
27. A kangaroo is sitting in the origin of a coordinate system. It can jump 1 unit vertically or horizontally. How many points are there in the plane at which the kangaroo can be after 10 jumps?
 A) 121 B) 100 C) 400 D) 441 E) Another answer
28. Let AD be a median in the triangle ABC . The angle ACB is 30° , the angle ADB is 45° . What is the measure of the angle BAD ?
 A) 45° B) 30° C) 25° D) 20° E) 15°
29. Find the minimal quantity of numbers one should remove from the set $\{1, 2, 3, \dots, 16\}$ so that the sum of any 2 remaining numbers were not a perfect square.
 A) 10 B) 9 C) 8 D) 7 E) 6
30. A prime number is defined as being *strange* if it is either a one-digit prime or if it has two or more digits, but both numbers obtained by omitting either its first or last digit are also strange. How many strange primes are there?
 A) 6 B) 7 C) 8 D) 9 E) 11



KANGAROO 2009



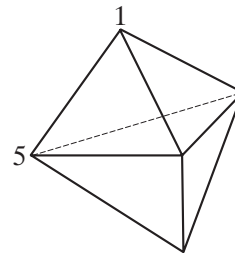
Student
11 and 12 grades

Time allowed: 75 min

Calculators are not permitted

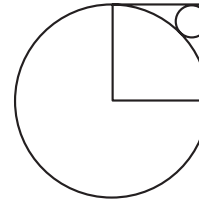
3-point questions

- There are 200 fishes in an aquarium. 1% of them is blue, all the rest are yellow. How many yellow fishes do we have to take out of the aquarium, so that the blue fishes represented 2% of all the aquarium fishes?
A) 2 B) 4 C) 20 D) 50 E) 100
- Which is the largest of the following numbers?
A) $\sqrt{2} - \sqrt{1}$ B) $\sqrt{3} - \sqrt{2}$ C) $\sqrt{4} - \sqrt{3}$ D) $\sqrt{5} - \sqrt{4}$ E) $\sqrt{6} - \sqrt{5}$
- For how many different positive integers n the number $n^2 + n$ is a prime number?
A) 0 B) 1 C) 2 D) More than 2, but a finite number E) An infinite number
- Mari, Ville and Ossi went to a café. Each of them bought three glasses of juice, two icecreams, and five buns. Which of the following sums could be the total bill?
A) 30.20Lt B) 29.20Lt C) 28.20Lt D) 27.20Lt E) 26.20Lt
- The picture shows a solid formed with 6 triangular faces. At each vertex there is a number. For each face we consider the sum of the 3 numbers at the vertices of that face. If all the sums are the same and two of the numbers are 1 and 5, as shown, what is the sum of all the 5 numbers?
A) 9 B) 12 C) 17 D) 18 E) 24
- Two circles of radii 13 and 15 intersect at points P and Q . Length of line segment PQ is 24. Which of the following numbers could be the distance between the centers of circles?
A) 13 B) 9 C) 5 D) 4 E) None of previous
- A box contains 2 white, 3 red and 4 blue socks. Jack knows that a third of the socks have a hole in them but he does not know what color the worn through socks are. In the darkness he takes some socks out of the box in a hope to find two good socks of the same color. How many socks must he take out to be absolutely sure to have a good pair?
A) 2 B) 3 C) 6 D) 7 E) 8

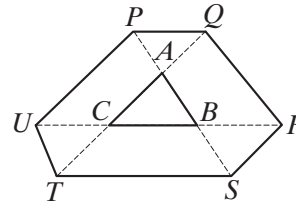


8. The square in the figure has a side equal to 1. Then the radius of the small circle is equal to

A) $\sqrt{2}-1$ B) $\frac{1}{4}$ C) $\frac{\sqrt{2}}{4}$ D) $1-\frac{\sqrt{2}}{2}$ E) $(1-\sqrt{2})^2$



9. The sides of triangle ABC are extended on both sides up to points P, Q, R, S, T and U so that $PA = AB = BS$, $TC = CA = AQ$ and $UC = CB = BR$. If the area of ABC is 1, what is the area of the hexagon $PQRSTU$?
A) 9 B) 10 C) 12 D) 13 E) 15



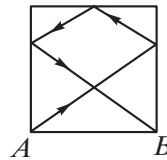
10. We want to color the squares in the grid using colors $a, b, c,$ and d so that neighboring squares were not of the same color (squares that share a vertex are considered neighbors). Some of the squares have been colored as shown. What are the possibilities for the shaded square?

| | | | | |
|-----|-----|-----|--|--|
| a | b | | | |
| c | d | | | |
| | | b | | |
| | | | | |
| b | | | | |

A) Only a or b B) Only c C) Only d D) Only c or d E) Any of a, b, c, d

4-point questions

11. On a square-shaped billiard table with a side 2 m long, a ball is thrown from the corner A . After touching three sides, as shown, it rolls to corner B . How many meters did the ball travel? (Remember that a ball bounces at the same angle as it enters, as shown in the picture on the right.)

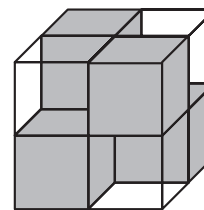


A) 7 B) $2\sqrt{13}$ C) 8 D) $4\sqrt{3}$ E) $2(\sqrt{2} + \sqrt{3})$

12. 2009 kangaroos, each of them either light or dark, compare their heights. It is known that one light kangaroo is taller than exactly 8 dark kangaroos, one light kangaroo is taller than exactly 9 dark kangaroos, one light kangaroo is taller than exactly 10 dark kangaroos, and so on, and exactly one light kangaroo is taller than all the dark kangaroos. What is the number of light kangaroos?

A) 1000 B) 1001 C) 1002 D) 1003 E) This situation is impossible

13. A cube measuring $2 \times 2 \times 2$ is formed from four $1 \times 1 \times 1$ white transparent and four $1 \times 1 \times 1$ black non-transparent cubes (see the picture). They are placed so that the whole big cube is non-transparent, meaning that it is not possible to see through it neither from top to bottom, nor from front to back and even not from left to right. How many black cubes at least should we have to put into the big cube measuring $3 \times 3 \times 3$ to make the whole cube non-transparent?



A) 6 B) 9 C) 10 D) 12 E) 18

14. 25 people are standing in a queue on the island of nobles and liars. Everyone, except the first person in the queue, said that the person ahead of him in the queue was a liar, and the first man in the queue said that all the people standing behind him were liars. How many liars were there in the queue? (Nobles always speak the truth, and liars always tell lies.)

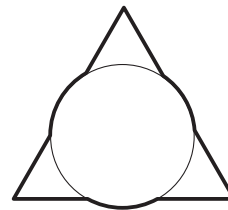
- A) 0 B) 12 C) 13 D) 24 E) Impossible to determine

15. What is the last digit of the number $1^2 - 2^2 + \dots - 2008^2 + 2009^2$?

- A) 1 B) 2 C) 3 D) 4 E) 5

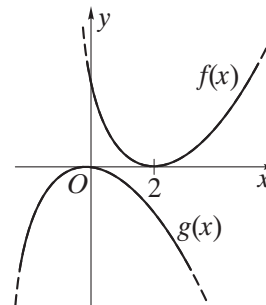
16. We overlap an equilateral triangle with the side length of 3 and a circle of radius 1 matching the centers of the two figures. How long is the perimeter of the figure that we get?

- A) $3 + 2\pi$ B) $6 + \pi$ C) $9 + \frac{\pi}{3}$ D) 3π E) $9 + \pi$



17. The graphs of real functions f and g are shown in the figure. What is the relation between f and g ?

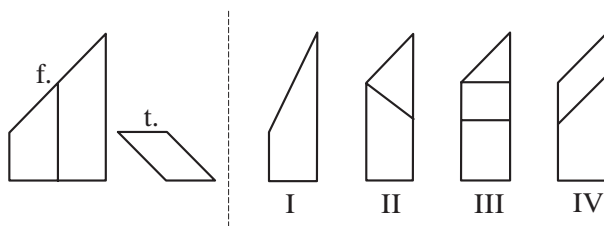
- A) $g(x) = f(x + 2)$
 B) $g(x - 2) = -f(x)$
 C) $g(x) = -f(-x + 2)$
 D) $g(-x) = -f(-x - 2)$
 E) $g(2 - x) = f(-x)$



18. Four problems were proposed to each of 100 contestants of a mathematical olympiad. 90 contestants solved the first problem, 85 contestants solved the second problem, 80 contestants solved the third problem, and 70 contestants solved the fourth problem. What is the smallest possible number of the contestants who solved all the four problems?

- A) 10 B) 15 C) 20 D) 25 E) 30

19. In the figure below, you see the front view (f.) and the top view (t.) of a geometric solid. Which of the figures from I to IV represents the view from the left?



- A) Figure I B) Figure II C) Figure III D) Figure IV E) None of them

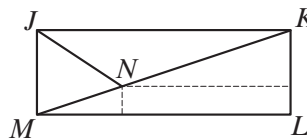
20. We have constructed a 3×3 -square table of real numbers in which the sum in each row, column, and diagonal is the same. Two of the numbers are shown in the figure. Which number is in the shaded square?

- A) 16 B) 51 C) 54 D) 55 E) 110

| | | |
|--|----|----|
| | | |
| | | 47 |
| | 63 | |

5-point questions

21. Two runners A and B are running round a stadium. A runs faster than B and it takes 3 minutes to A for one lap. A and B started together, and 8 minutes later A caught B for the first time. How long does it take for B to run one lap?
 A) 6 min B) 8 min C) 4 min 30 s D) 4 min 48 s E) 4 min 20 s
22. Let m be the number of 8-digit numbers with 8 different digits, none of which is 0. How many of them are divisible by 9?
 A) $\frac{m}{8}$ B) $\frac{m}{3}$ C) $\frac{m}{9}$ D) $\frac{8m}{9}$ E) $\frac{7m}{8}$
23. How many ten-digit numbers, composed only of digits 1, 2 and 3, do there exist, in which any two neighboring digits differ by 1?
 A) 16 B) 32 C) 64 D) 80 E) 100
24. For how many integers $n \geq 3$ does there exist a convex n -gon whose angles in some order are in ratio $1 : 2 : \dots : n$?
 A) 1 B) 2 C) 3 D) 5 E) More than 5
25. 55 schoolchildren took part in math olympiad. When checking the problems, the jury marked them either by „+“ if the problem was solved, or by „-“ if the problem was not solved, or by „0“ if participant skipped the problem. Later it turned out that no two papers had the same number of „+“ and „-“. What is the least number of problems at the olympiad?
 A) 6 B) 9 C) 10 D) 11 E) 12
26. In a rectangle $JKLM$, the bisector of angle KJM cuts the diagonal KM at point N . The distances between N and sides LM and KL are 1 and 8, respectively. Then LM is:
 A) $8 + 2\sqrt{2}$ B) $11 - \sqrt{2}$ C) 10 D) $8 + 3\sqrt{2}$ E) $11 + \frac{\sqrt{2}}{2}$



27. If $k = \frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b}$, how many possible values of k are there?
 A) 1 B) 2 C) 3 D) 4 E) 6
28. The numbers $1, 2, 3, \dots, 99$ are distributed into n groups (at least 2 numbers in a group) under the condition:
if two numbers are in one and the same group, then their sum is not divisible by 3.
 The smallest n with this property is:
 A) 3 B) 9 C) 33 D) 34 E) 66
29. Man Friday wrote down in a row several different integers smaller than 11. Robinson Crusoe examined these numbers and noticed with satisfaction that in each pair of the neighbouring numbers one of the numbers was divisible by another. How many numbers at most could Man Friday write down?
 A) 6 B) 7 C) 8 D) 9 E) 10
30. The sequence of integers a_n is defined by: $a_0 = 1, a_1 = 2, a_{n+2} = a_n + (a_{n+1})^2$ for $n \geq 0$. The remainder in the division of a_{2009} by 7 is:
 A) 0 B) 1 C) 2 D) 5 E) 6