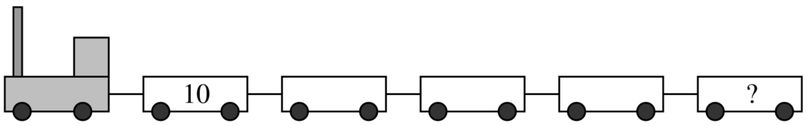
Questions of Kangaroo 2003

MINOR (grades 3 and 4)

3-POINT QUESTIONS

M1. How much is $0+1+2+3+4-3-2-1-0$?
A 0 **B** 2 **C** 4 **D** 10 **E** 16

M2. There are 10 boxes in the first van. Every further van contains twice as many boxes as the previous one. How many boxes are there in the fifth van?



A 100 **B** 120 **C** 140 **D** 160 **E** 180

M3. Sophie draws kangaroos: a blue one, then a green, then a red, then a black, then a yellow, a blue, a green, a red, a black, and so on. What colour is the 17$^{\text{th}}$ kangaroo?
A Blue **B** Green **C** Red **D** Black **E** Yellow

M4. In the teachers' room there are 6 tables with 4 chairs each, 4 tables with 2 chairs each, and 3 tables with 6 chairs each. How many chairs are there altogether?
A 40 **B** 25 **C** 50 **D** 36 **E** 44

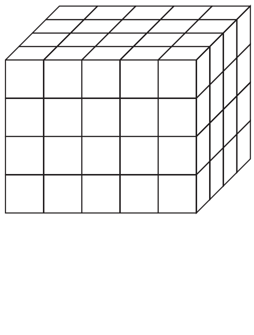
M5. A coin is lying on the table. What is the maximum number of such coins which can be put on the table in such a way that each of them touches this coin?
A 4 **B** 5 **C** 6 **D** 7 **E** 8

M6. In the picture the distance $KM=10$, $LN=15$, $KN=22$. Find the distance LM.

A 1 **B** 2 **C** 3 **D** 4 **E** 5

M7. Hedgehog Mark complained to his friends: "If I had picked twice as many apples as I really did, I would have 24 apples more than I have now." How many apples did Mark pick?
A 48 **B** 24 **C** 42 **D** 12 **E** 36

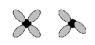
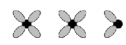
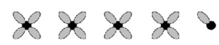
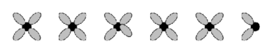
M8. Chris constructed the brick on the picture using red and blue cubes of the same size. The outside of the brick is completely red, but all cubes used inside are blue. How many blue cubes did Chris use?
A 12 **B** 24 **C** 36 **D** 40 **E** 48



4-POINT QUESTIONS

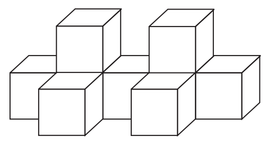
M9. A rectangle of size 4×7 is drawn on a squared sheet of paper. How many squares of the size 1×1 are cut into two parts by the diagonal of that rectangle?
A 8 **B** 9 **C** 10 **D** 11 **E** 12

M10. This table shows the quantity of different types of flowers in the botanical garden. Ted was told by the gardener that there were 35 azaleas, 50 irises and 85 roses in the garden. What is the number of gerberas growing in the garden?
A 95 **B** 100 **C** 105 **D** 110 **E** 115

Azaleas	
Irises	
Roses	
Gerberas	

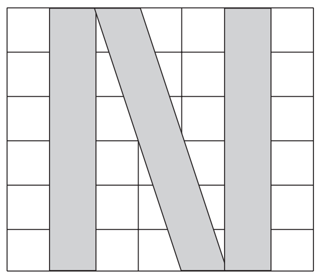
M11. Annie fell asleep at 9:30 pm and woke up at 6:45 am. Her brother Martin slept 1 hour 50 min longer. How many hours and minutes did Martin sleep?
A 30 h 5 min **B** 11 h 35 min **C** 11 h 5 min **D** 9 h 5 min **E** 8 h 35 min

M12. The construction in the picture is built of cubes of the same size and weighs 189 grams. How many grams does one cube weigh?
A 29 **B** 25 **C** 21 **D** 19 **E** 17



M13. Kangaroo Jumpy was training for the Animal Olympiad. His longest jump during the training was 50 dm 50 cm 50 mm long. In the end he won the gold medal at the Olympiad with a jump that was 123 cm longer. How long was Jumpy's winning jump?
A 6 m 78 cm **B** 5 m 73 cm **C** 5 m 55 cm **D** 11 m 28 cm **E** 7 m 23 cm

M14. If the length of the side of a little square is 1, what is the area of the letter N?
A 15 **B** 16 **C** 17 **D** 18 **E** 19



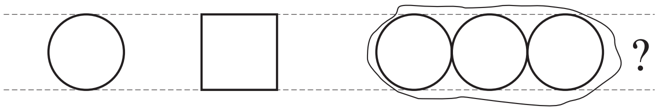
M15. Betty likes calculating the sum of the digits that she sees on her digital clock (for instance, if the clock shows 21:37, then Betty gets $2 + 1 + 3 + 7 = 13$). What is the maximum sum she can get?
A 24 **B** 36 **C** 19 **D** 25 **E** 23

M16. In the class there are 29 children. 12 children have a sister and 18 children have a brother. Tina, Bert, and Ann have no brother and no sister. How many children in that class have both a brother and sister?
A No one **B** 1 **C** 3 **D** 4 **E** 6

5-POINT QUESTIONS

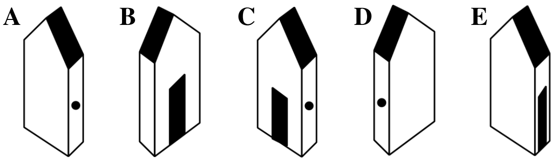
M17. Joe wants to buy some balls. If he bought five balls, he would still have 10 dollars left in his wallet. If he wanted to buy seven balls, he would have to borrow 22 dollars. How much is one ball (the price of a ball is an integer number)?
A 11 **B** 16 **C** 22 **D** 26 **E** 32

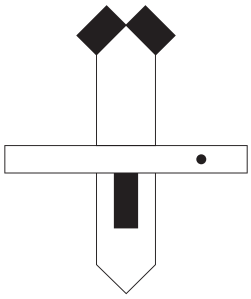
M18. I surrounded the wooden circle (see picture) using a cm of thread. After that I surrounded by thread the wooden square — b cm of thread was enough for that. How much thread (in cm) would be enough to surround the three wooden circles without moving them?



A $3a$ **B** $2a+b$ **C** $a+2b$ **D** $3b$ **E** $a+b$

M19. The picture on the right has been drawn on paper and cut out to make a house. Which of the houses does it make?

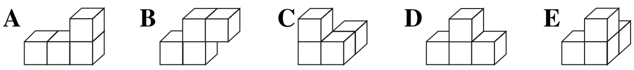


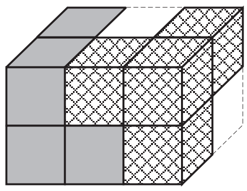


M20. Kangaroo bought 3 types of sweets: big, medium and small ones. Big sweets cost 4 coins per 1, medium — 2 coins per 1, small — 1 coin per 1. Kangaroo bought 10 sweets and he paid 16 coins. How many big sweets did kangaroo buy?
A 5 **B** 4 **C** 3 **D** 2 **E** 1

M21. A bar-code is formed by 17 alternating black and white bars (the first and the last bars are black). The black bars are of two types: wide and narrow. The number of white bars is greater by 3 than the number of wide black bars. Then the number of narrow black bars is
A 1 **B** 2 **C** 3 **D** 4 **E** 5

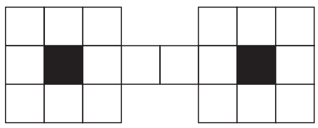
M22. A rectangular parallelepiped was composed of 3 pieces, each consisting of 4 little cubes. Then one piece was removed (see picture). Which one?





M23. In the toy shop the price for one dog and three bears is the same as for four kangaroos. Three dogs and two bears together also have the same price as four kangaroos. What is more expensive and how many times — the dog or the bear?
A The dog is two times more expensive
B The bear is two times more expensive
C The same price
D The bear is three times more expensive
E The dog is three times more expensive

M24. The composite board shown in the picture consists of 20 fields 1×1. How many possibilities are there to cover all 18 white fields with 9 rectangular stones 1×2? (The board cannot be turned. Two possibilities are different if at least one stone lies in another way.)
A 2 **B** 4 **C** 6 **D** 8 **E** 16



BENJAMIN (grades 5 and 6)

3-POINT QUESTIONS

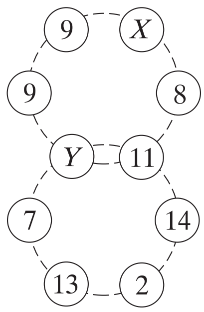
B1. Which number is the greatest?
A $2+0+0+3$ **B** $2\times0\times0\times3$ **C** $(2+0)\times(0+3)$ **D** $20\times0\times3$ **E** $(2\times0)+(0\times3)$

B2. Sophie draws kangaroos: a blue one, then a green, then a red, then a black, then a yellow, a blue, a green, a red, a black, and so on. What colour is the 17$^{\text{th}}$ kangaroo?
A Blue **B** Green **C** Red **D** Black **E** Yellow

B3. How many integers can one find in the interval from 2.09 to 15.3?
A 13 **B** 14 **C** 11 **D** 12 **E** Infinitely many

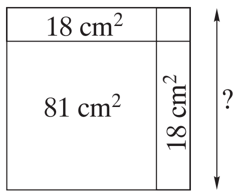
B4. Which is the smallest positive integer divisible by 2, 3, and 4?
A 1 **B** 6 **C** 12 **D** 24 **E** 36

B5. The sum of the numbers in each of the rings should be 55. Which number is X?
A 9 **B** 10 **C** 13 **D** 16 **E** 18



B6. Tom has 9 banknotes of 100 dollars each, 9 banknotes of 10 dollars each, and 10 banknotes of 1 dollar each. How many dollars has he?
A 1 000 **B** 991 **C** 9 910 **D** 9 901 **E** 99 010

B7. The square in the picture consists of two smaller squares and two rectangles of area 18 cm^2 each. The area of one of smaller rectangles is 81 cm^2. What is the length (in cm) of side of the biggest square?
A 9 **B** 2 **C** 7 **D** 11 **E** 10



B8. Betty likes calculating the sum of the digits that she sees on her digital clock (for instance, if the clock shows 21:37, then Betty gets $2+1+3+7=13$). What is the maximum sum she can get?
A 24 **B** 36 **C** 19 **D** 25 **E** 23

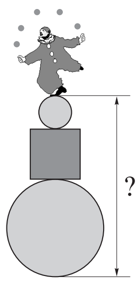
B9. In the picture the distance $KM=10$, $LN=15$, $KN=22$. Find the distance LM.

A 1 **B** 2 **C** 3 **D** 4 **E** 5

B10. The number 24 has eight divisors: 1, 2, 3, 4, 6, 8, 12 and 24. Find the smallest number having four divisors.
A 4 **B** 6 **C** 8 **D** 9 **E** 10

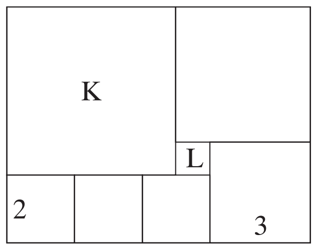
4-POINT QUESTIONS

B11. The picture shows the clown Dave dancing on top of two balls and one cubic box. The radius of the lower ball is 6 dm, the radius of the upper ball is three times less. The side of the cubic box is 4 dm longer than the radius of the upper ball. At what height (in dm) above the ground is the clown Dave standing?
A 14 **B** 20 **C** 22 **D** 24 **E** 28



B12. We take two different numbers from 1, 2, 3, 4, 5 and find their sum. How many different sums can we obtain?
A 5 **B** 6 **C** 7 **D** 8 **E** 9

B13. The rectangle in the picture consists of 7 squares. The lengths of the sides of some of the squares are shown. Square K is the biggest one, square L — the smallest one. How many times is the area of K bigger than the area of L?
A 16 **B** 25 **C** 36 **D** 49 **E** Impossible to find



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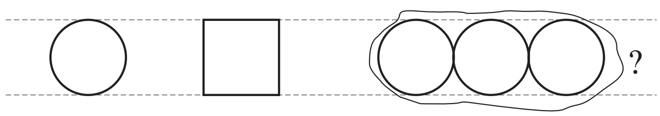
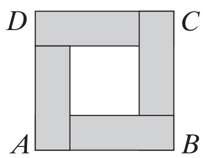


A $3a$ **B** $2a+b$ **C** $a+2b$ **D** $3b$ **E** $a+b$

B15. Benito has 20 small balls of different colours: yellow, green, blue and black. 17 of the balls are not green, 5 are black, 12 are not yellow. How many blue balls does Benito have?
A 3 **B** 4 **C** 5 **D** 8 **E** 15

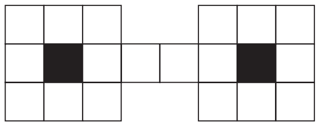
B16. There are 17 trees along the road from Basil's home to a pool. Basil marked some trees with a red strip as follows. On his way to bathe he marked the first tree and then each second tree, and on his way back he marked the first tree and then each third tree. How many trees have no mark after that?
A 4 **B** 5 **C** 6 **D** 7 **E** 8

B17. Square $ABCD$ is comprised of one inner square (white) and four shaded congruent rectangles. Each shaded rectangle has a perimeter of 40 cm. What is the area (in cm^2) of square $ABCD$?
A 400 **B** 200 **C** 160 **D** 100 **E** 80



B18. Today's date is 20.03.2003. What date will it be 2003 minutes after the hour 20:03?
A 21.03.2003 **B** 22.03.2003 **C** 23.03.2003 **D** 21.04.2003 **E** 22.04.2003

B19. The composite board shown in the picture consists of 20 fields 1×1. How many possibilities are there exist to cover all 18 white fields with 9 rectangular stones 1×2? (The board cannot be turned. Two possibilities are called different if at least one stone lies in another way.)



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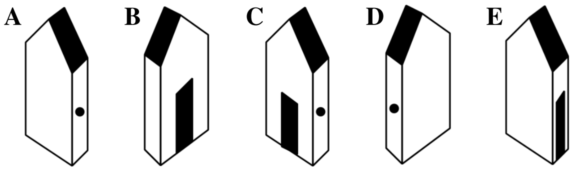
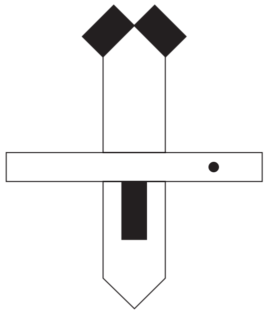
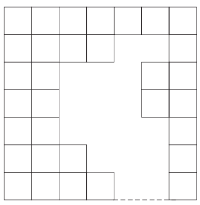
5-POINT QUESTIONS

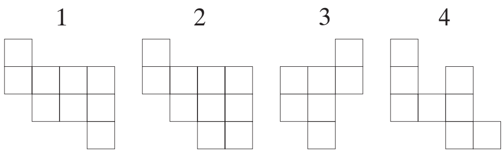
B21. The picture on the right has been drawn on paper and cut out to make a house. Which of the houses does it make?





B22. The square was cut out from a page in a squared exercise book. Then two figures in the picture were cut out from the square. Which ones?

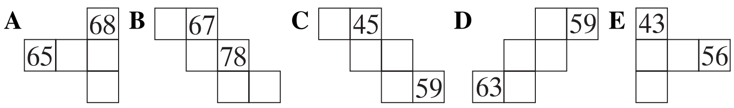




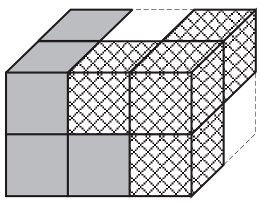
A 1 and 3 **B** 2 and 4 **C** 2 and 3 **D** 1 and 4 **E** Impossible to cut out

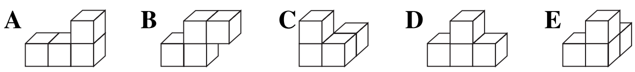
B23. Walter displayed all the integers from 0 to 109 according to some simple rule. Here is the beginning of his 5-column numeral chart. Which of the following elements could not be the a part of Walter's chart?

0	2	4	6	8
1	3	5	7	9
10	12	14	16	18
11	13	15	17	19
20	22	24	26	28



B24. A rectangular parallelepiped was composed of 3 pieces, each consisting of 4 little cubes. Then one piece was removed (see picture). Which one?

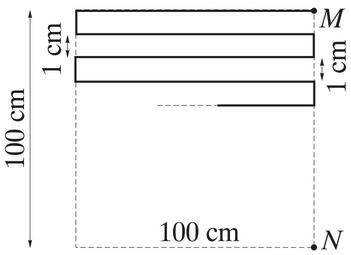




B25. You have six line segments of lengths 1 cm, 2 cm, 3 cm, 2001 cm, 2002 cm and 2003 cm. You have to choose three of these segments to form a triangle. How many different choices of three segments are there which work?
A 1 **B** 3 **C** 5 **D** 6 **E** More than 10

B26. There were completely red and completely green dragons in the dungeon. Each red dragon had 6 heads, 8 legs and 2 tails. Each green dragon had 8 heads, 6 legs and 4 tails. In all the dragons had 44 tails. The number of green legs was 6 fewer than of red heads. How many red dragons were there in the dungeon?
A 6 **B** 7 **C** 8 **D** 9 **E** 10

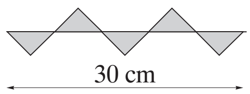
B27. What is the length (in cm) of the line (see picture) connecting vertices M and N of the square?
A 10 200 **B** 2 500 **C** 909 **D** 10 100 **E** 9 900



B28. Every figure in the picture replaces some digit. What is the sum □ + ◯?
A 6 **B** 7 **C** 8 **D** 9 **E** 13

□□□
+□□◯
□△△
2 0 0 3

B29. The figure in the drawing consists of five isosceles right triangles of the same size. Find the area (in cm^2) of the shaded figure.
A 20 **B** 25 **C** 35 **D** 45 **E** Cannot be found



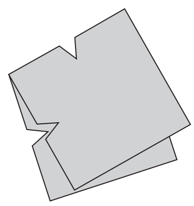
B30. Ann has the box containing 9 pencils. At least one of them is blue. Among every 4 of the pencils at least two have the same colour, and among every 5 of the pencils at most three have the same colour. What is the number of blue pencils?
A 2 **B** 3 **C** 4 **D** 1 **E** Impossible to determine

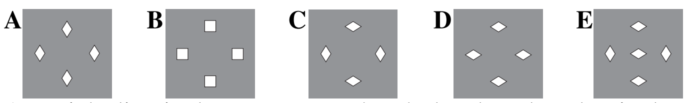
CADET (grades 7 and 8)

3-POINT QUESTIONS

C1. There were 5 parrots in a pet shop. Their average price was 6 000 dollars. One day the most expensive parrot was sold. The average price of the remaining four parrots was 5 000 dollars. What was the price (in dollars) of the parrot sold?
A 1 000 **B** 2 000 **C** 5 500 **D** 6 000 **E** 10 000

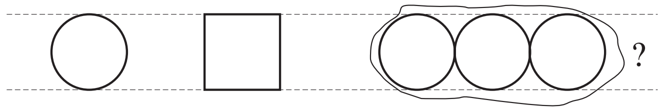
C2. A folded napkin was cut through (see picture). What does it look like when unfolded?





C3. A straight line is drawn across a 4×4 chessboard. What is the greatest number of 1×1 squares which can be cut into two pieces by the line?
A 3 **B** 4 **C** 6 **D** 7 **E** 8

C4. The area of the wooden square equals a. The area of each wooden circle equals b. Three circles are lined up as shown in the picture. If we tie together the three circles with a thread as short as possible, without moving them, what is the area inside the thread?

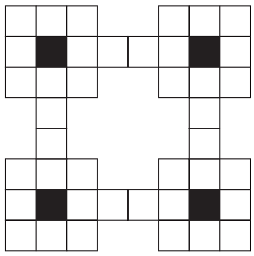


A $3b$ **B** $2a + b$ **C** $a + 2b$ **D** $3a$ **E** $a + b$

C5. For a hexagon (not necessarily convex), the maximum possible number of interior right angles is
A 2 **B** 3 **C** 4 **D** 5 **E** 6

C6. A bottle and a glass together have the same volume as a jug. A bottle has the same volume as a glass and a tankard. Three tankards have the same volume as two jugs. How many glasses of water equal one tankard?
A 3 **B** 4 **C** 5 **D** 6 **E** 7

C7. The composite board shown in the picture consists of 44 fields 1×1. How many possibilities are there to cover all 40 white fields with 20 rectangular stones 1×2? (The board cannot be turned. Two possibilities are different if at least one stone lies in another way.)
A 8 **B** 16 **C** 32 **D** 64 **E** 100



C8. In a positive integer consisting of at least 2 digits, the last digit has been crossed out, so that the number has been decreased n times. What is the maximum value of n?
A 9 **B** 10 **C** 11 **D** 19 **E** 20

C9. There are four line segments drawn. Which number of intersection points *is impossible*?
A 2 **B** 3 **C** 5 **D** 6 **E** 7

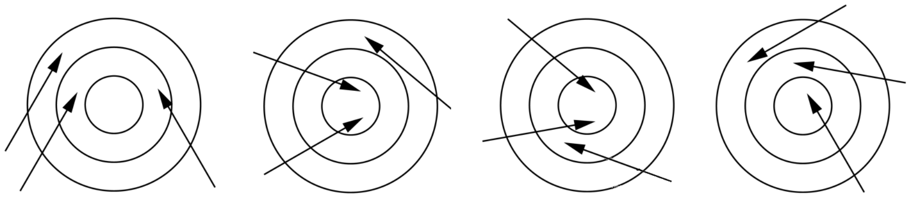
C10. Which of the following numbers gives, when multiplied by 768, the product ending with the highest number of zeroes?
A 7 500 **B** 5 000 **C** 3 125 **D** 2 500 **E** 10 000

4-POINT QUESTIONS

C11. Lying on a table, there is a transparent square sheet of film with the letter y written on it. We turn the sheet 90° clockwise, then turn it over from its right side, then turn it 180° counterclockwise. What do we now see?
A  **B**  **C**  **D**  **E** 

C12. Mike has 42 identical cubes, each with the edge 1 cm long. He used all of the cubes to construct a cuboid. The perimeter of the base of that cuboid is 18 cm. What is its height?
A 1 cm **B** 2 cm **C** 3 cm **D** 4 cm **E** 5 cm

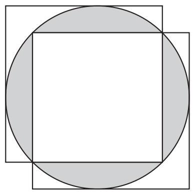
C13. Jeffrey shoots three arrows at each of four identical targets. He scores 29 points on the first target, 43 on the second and 47 on the third. How many points does Jeffrey score on the last target?



A 31 **B** 33 **C** 36 **D** 38 **E** 39

C14. The weight of a truck without a load is 2000 kg. Today the load initially comprised 80% of the total weight. At the first stop, a quarter of the load was left. What percentage of the total weight does the load then comprise?
A 20% **B** 25% **C** 55% **D** 60% **E** 75%

C15. Two quadrates with the same size cover a circle, the radius of which is 3 cm. Find the total area (in cm^2) of the shaded figure.



A $8(\pi - 1)$ **B** $6(2\pi - 1)$ **C** $9\pi - 25$ **D** $9(\pi - 2)$ **E** $\frac{6\pi}{5}$

C16. You have six line segments of lengths 1 cm, 2 cm, 3 cm, 2001 cm, 2002 cm and 2003 cm. You have to choose three of these segments to form a triangle. How many different choices of three segments are there which work?
A 1 **B** 3 **C** 5 **D** 6 **E** More than 10

C17. How many positive integers n possess the following property: among the positive divisors of n different from 1 and n itself, the largest is 15 times the smallest.
A 0 **B** 1 **C** 2 **D** 3 **E** Infinitely many

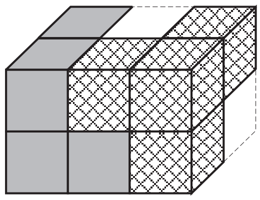
C18. Six points K, L, M, N, P, R are marked on a line from left to right, in the same order as listed. It is known that $KN = MR$ and $LN = NR$. Then, necessarily
A $KL = LM$ **B** $LM = NP$ **C** $LN = PR$ **D** $KL = MN$ **E** $MN = PR$

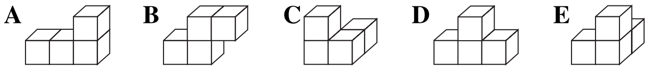
C19. Mary has 6 cards with natural numbers written on them (one number on each card). She chooses 3 cards and calculates the sum of the corresponding numbers. Having done this for all 20 possible combinations of 3 cards, she discovers that 10 sums are equal to 16, and the other 10 sums are equal to 18. Then the smallest number on the cards is
A 2 **B** 3 **C** 4 **D** 5 **E** 6

C20. Paul, Bill, John, Nick and Tim stood in a circle, the distances between any two neighbours being different. Each of them said the name of the boy standing closest to him. The names Paul and Bill were said two times each, and the name John was said once. Then
A Paul and Bill were not neighbours
B Nick and Tim were not neighbours
C Nick and Tim were neighbours
D The situation described is impossible
E None of the above

5-POINT QUESTIONS

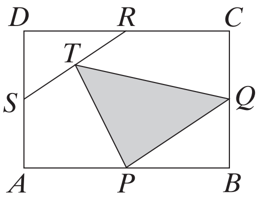
C21. A rectangular parallelepiped was composed of 3 pieces, each consisting of 4 little cubes. Then one piece was removed (see picture). Which one?





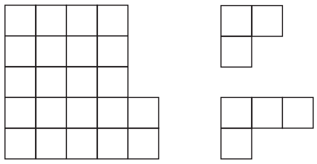
C22. In a rectangle $ABCD$, let P, Q, R and S be the midpoints of sides AB, BC, CD and AD, respectively, and let T be the midpoint of segment RS. Which fraction of the area of $ABCD$ does triangle PQT cover?

A $\frac{5}{16}$ **B** $\frac{1}{4}$ **C** $\frac{1}{5}$ **D** $\frac{1}{6}$ **E** $\frac{3}{8}$



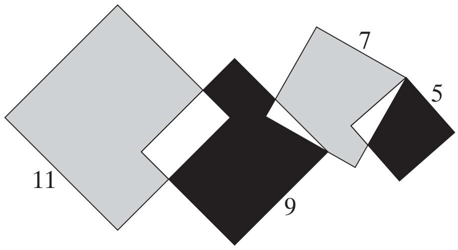
C23. Carl composed the figure shown on the left side of the drawing from the smaller three-square and four-square figures shown on the right side. The smaller figures can be turned around, but not turned over. What is the smallest number of three-square figures needed for that?

A 1 **B** 2 **C** 3 **D** 6 **E** Impossible to compose



C24. In the picture there are four overlapping squares with sides 11, 9, 7 and 5 long. How much greater is the sum of the two grey areas than the sum of the two black areas?

A 25 **B** 36 **C** 49 **D** 64 **E** 0

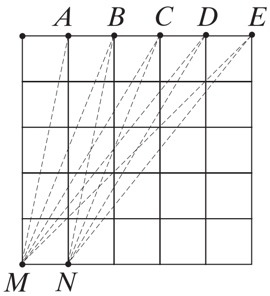


C25. On a bookshelf there are 50 math and physics books. No two physics books stand side by side, but every math book has a math neighbour. Which of the following statements *may turn out to be false*?

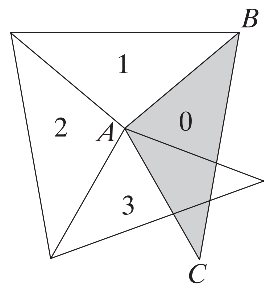
A The number of math books is at least 32
B The number of physics books is at most 17
C There are 3 math books standing in succession
D If the number of physics books is 17, then at least one of them is the first or the last on the bookshelf
E Among any 9 successive books, at least 6 are math books

C26. A square is divided into 25 small squares (see the picture). Find the measure of the angle which is the sum of the angles MAN, MBN, MCN, MDN, MEN.

A 30° **B** 45° **C** 60° **D** 75° **E** 90°



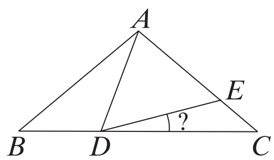
C27. We are going to make a spiral of isosceles triangles. We'll start with the shaded triangle BAC, which has a top angle $\angle BAC = 100^\circ$, and move counterclockwise. Let ΔABC have number 0. Every of the next triangles (with numbers 1, 2, 3,...) will have exactly one edge adjoining the previous one (see the picture). What will be the number of the first triangle which precisely covers triangle nr. 0?
A 10 **B** 12 **C** 14 **D** 16 **E** 18



C28. How many positive integers n can be found such that 2003 divided by n leaves a remainder of 23?
A 22 **B** 19 **C** 13 **D** 12 **E** 36

C29. There are some 10 points on the area, and there are no three points on the same line. Every two points are connected by a segment. What is the largest possible number of these segments, which can be crossed by another line that doesn't pass through any of these points?
A 20 **B** 25 **C** 30 **D** 35 **E** 45

C30. In triangle ABC (see picture) $AB = AC$, $AE = AD$, and $\angle BAD = 30^\circ$. What is the measure of angle CDE?
A 10° **B** 15° **C** 20° **D** 25° **E** 30°

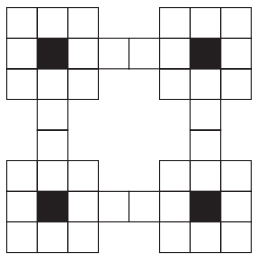


JUNIOR (grades 9 and 10)

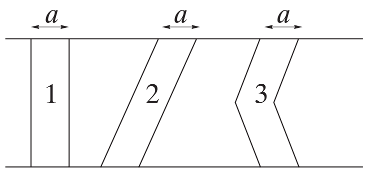
3-POINT QUESTIONS

J1. 15% of a round cake is cut as shown in the figure. How many degrees is the angle denoted by the question mark?
A 30° **B** 45° **C** 54° **D** 15° **E** 20°

J2. The composite board shown in the picture consists of 44 fields 1×1. How many possibilities are there to cover all 40 white fields with 20 rectangular stones 1×2? (The board cannot be turned. Two possibilities are different if at least one stone lies in another way.)
A 8 **B** 16 **C** 32 **D** 64 **E** 100



J3. In the picture, three strips of the same horizontal width a are marked 1, 2, 3. These strips connect the two parallel lines. Which strip has the biggest area?
A All three strips have the same area
B Strip 1 **C** Strip 2 **D** Strip 3
E Impossible to answer without knowing a



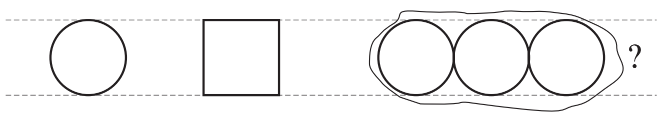
J4. Which of the following numbers is odd for every integer n?
A $2003n$ **B** $n^2 + 2003$ **C** n^3 **D** $n + 2004$ **E** $2n^2 + 2003$

J5. In a triangle ABC the angle C is three times bigger than the angle A, the angle B is two times bigger than the angle A. Then the triangle ABC
A is equilateral **B** is isosceles **C** has an obtuse angle **D** has a right angle
E has only acute angles

J6. Three singers take part in a musical round of 4 equal lines, each finishing after singing the round 3 times. The second singer begins the first line when the first singer begins the second line, the third singer begins the first line when the first singer begins the third line. The fraction of the total singing time that all three are singing at the same time is
A $\frac{3}{5}$ **B** $\frac{4}{5}$ **C** $\frac{4}{7}$ **D** $\frac{5}{7}$ **E** $\frac{7}{11}$

J7. The number $a = 111\ldots111$ consists of 2003 digits, each equal to 1. What is the sum of the digits of the product $2003 \cdot a$?
A 10 000 **B** 10 015 **C** 10 020 **D** 10 030 **E** 2003^2

J8. The area of the wooden square equals a. The area of each wooden circle equals b. Three circles are lined up as shown in the picture. If we tie together the three circles with a thread as short as possible, without moving them, what is the area inside the thread?



A $3b$ **B** $2a + b$ **C** $a + 2b$ **D** $3a$ **E** $a + b$

J9. How many of the functions $f(x) = 0,\ f(x) = \frac{1}{2},\ f(x) = 1,\ f(x) = x,\ f(x) = -x$ satisfy the equation $f(x^2 + y^2) = f^2(x) + f^2(y)$?
A 1 **B** 2 **C** 3 **D** 4 **E** 5

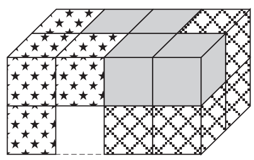
J10. In this addition each of the letters X, Y and Z represents a different non-zero digit. The letter X will then have to stand for
A 1 **B** 2 **C** 7 **D** 8 **E** 9

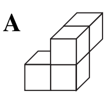
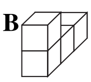
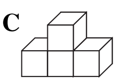
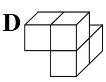
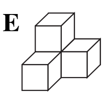
$$\begin{array}{r} XX \\ +\,YY \\ ZZ \\ \hline ZYX \end{array}$$

4-POINT QUESTIONS

J11. Ann has a box containing 9 pencils. At least one of them is blue. Among every 4 of the pencils at least two have the same colour, and among every 5 of the pencils at most three have the same colour. What is the number of blue pencils?
A 2 **B** 3 **C** 4 **D** 1 **E** Impossible to determine

J12. A rectangular parallelepiped was composed of 4 pieces, each consisting of 4 little cubes. Then one piece was removed (see picture). Which one?

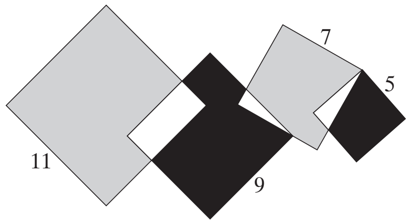


A  **B**  **C**  **D**  **E** 

J13. When a barrel is 30% empty it contains 30 litres more when it is 30% full. How many litres does the barrel hold when full?
A 60 **B** 75 **C** 90 **D** 100 **E** 120

J14. Each of two pupils changed two of the digits of 3-digit number 888 and got a new 3-digit number which is still divisible by 8. What is the biggest possible difference of their numbers?
A 800 **B** 840 **C** 856 **D** 864 **E** 904

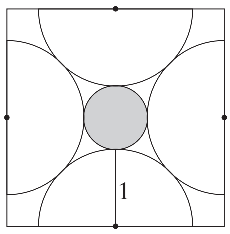
J15. In the picture there are four overlapping squares with sides 11, 9, 7 and 5 long. How much greater is the sum of the two grey areas than the sum of the two black areas?
A 25 **B** 36 **C** 49 **D** 64 **E** 0



J16. The value of the product

$$\left(1+\frac{1}{2}\right)\cdot\left(1+\frac{1}{3}\right)\cdot\left(1+\frac{1}{4}\right)\cdots\left(1+\frac{1}{2003}\right)$$

is equal to
A 2004 **B** 2003 **C** 2002 **D** 1002 **E** 1001



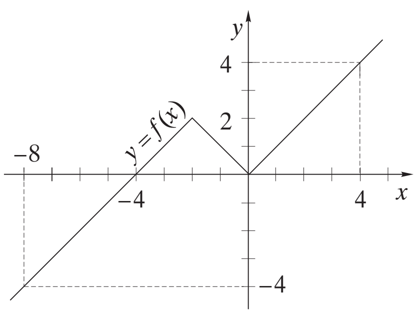
J17. The diagram shows four semicircles with radius 1. The centres of the semicircles are at the mid-points of the sides of a square. What is the radius of the circle which touches all four semicircles?

A $\sqrt{2}-1$ **B** $\frac{\pi}{2}-1$ **C** $\sqrt{3}-1$ **D** $\sqrt{5}-2$ **E** $\sqrt{7}-2$

J18. Consider all the different four-digit numbers that you can form by using the four digits of the number 2003. Summing up all them (including 2003 itself) you get:
A 5 005 **B** 5 555 **C** 16 665 **D** 1 110 **E** 15 555

J19. The first two terms of a sequence are 1 and 2. Each next term is obtained by dividing the term before the previous one by the previous term. What is the tenth term of this sequence?
A 2^{-10} **B** 256 **C** 2^{-13} **D** 1024 **E** 2^{34}

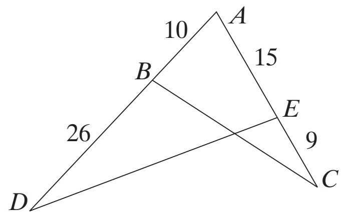
J20. The graph of the function $f(x)$, defined for all real numbers, is formed by two half-lines and one segment, as illustrated in the picture.



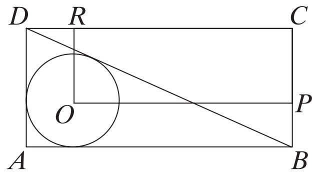
Clearly, -8 is a solution of the equation $f(f(x)) = 0$, because $f(f(-8)) = f(-4) = 0$. Find all the solutions of the equation $f(f(f(x))) = 0$.
A -4; 0 **B** -8; -4; 0 **C** -12; -8; -4; 0 **D** -16; -12; -8; -4; 0 **E** No solutions

5-POINT QUESTIONS

J21. What is the ratio of the areas of the triangles ADE and ABC in the picture?
A $\frac{9}{4}$ **B** $\frac{7}{3}$ **C** $\frac{4}{5}$ **D** $\frac{15}{10}$ **E** $\frac{26}{9}$

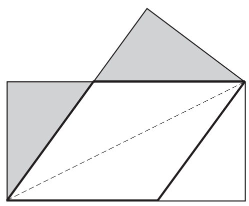


J22. The rectangle $ABCD$ has area 36. A circle with center in point O is inscribed in the triangle ABD. What is the area of the rectangle $OPCR$?
A 24 **B** 6π **C** 18 **D** $12\sqrt{2}$
E It depends on the ratio of the sides AB and AD



J23. The children K, L, M and N made the following assertions:
K: L, M and N are girls; L: K, M and N are boys;
M: K and L are lying; N: K, L and M are telling the truth.
How many of the children were telling the truth?
A 0 **B** 1 **C** 2 **D** 3 **E** Impossible to determine

J24. A rectangular sheet of paper with measures 6×12 is folded along its diagonal. The shaded parts sticking out over the edge of the overlapping area are cut off and the sheet is unfolded. Now it has the shape of a rhombus. Find the length of the side of the rhombus.
A $\frac{7\sqrt{5}}{2}$ **B** 7.35 **C** 7.5 **D** 7.85 **E** 8.1



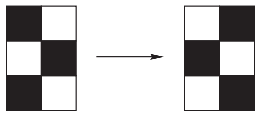
J25. How many distinct pairs $(x; y)$ satisfy the equation $(x + y)^2 = xy$?
A 0 **B** 1 **C** 2 **D** 3 **E** Infinitely many

J26. What is the greatest number of consecutive integers such that the sum of the digits of none of them is divisible by 5?
A 5 **B** 6 **C** 7 **D** 8 **E** 9

J27. On a bookshelf there are 50 math and physics books. No two physics books stand side by side, but every math book has a math neighbour. Which of the following statements *may turn out to be false*?
A The number of math books is at least 32
B The number of physics books is at most 17
C There are 3 math books standing in succession
D If the number of physics books is 17, then at least one of them is the first or the last on the bookshelf
E Among any 9 successive books, at least 6 are math books

J28. We take three different numbers from the numbers 1, 4, 7, 10, 13, 16, 19, 22, 25, 28 and find their sum. How many different sums can we obtain?
A 13 **B** 21 **C** 22 **D** 30 **E** 120

J29. Unit squares of a squared board 2×3 are coloured black and white like a chessboard (see picture). Determine the minimum number of steps necessary to achieve the reverse of the left board, following the rule: in each step, we must repaint two unit squares that have a joint edge, but we must repaint a black square with green, a green square with white and a white square with black.



A 3 **B** 5 **C** 6 **D** 8 **E** 9

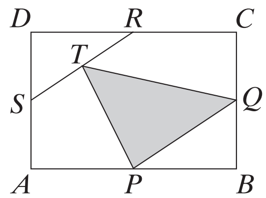
J30. We wrote down all the integers of 1 to 5 digits we could, using only the two digits 0 and 1. How many 1's did we write?
A 36 **B** 48 **C** 80 **D** 160 **E** 320

STUDENT (grades 11 and 12)

3-POINT QUESTIONS

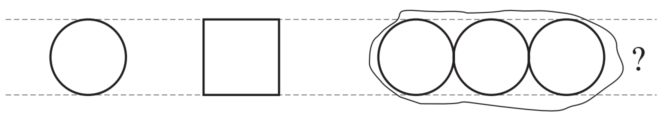
S1. Ann has a box containing 9 pencils. At least one of them is blue. Among every 4 of the pencils at least two have the same colour, and among every 5 of the pencils at most three have the same colour. What is the number of blue pencils?
A 2 **B** 3 **C** 4 **D** 1 **E** Impossible to determine

S2. In a rectangle $ABCD$, let P, Q, R and S be the midpoints of sides AB, BC, CD and AD, respectively, and let T be the midpoint of segment RS. Which fraction of the area of $ABCD$ does triangle PQT cover?



A $\frac{5}{16}$ **B** $\frac{1}{4}$ **C** $\frac{1}{5}$ **D** $\frac{1}{6}$ **E** $\frac{3}{8}$

S3. The area of the wooden square equals a. The area of each wooden circle equals b. Three circles are lined up as shown in the picture. If we tie together the three circles with a thread as short as possible, without moving them, what is the area inside the thread?



A $3b$ **B** $2a + b$ **C** $a + 2b$ **D** $3a$ **E** $a + b$

S4. Alan was calculating the volume of a sphere, but in the calculation he mistakenly used the value of the diameter instead of the radius of the sphere. What should he do with his result to get the correct answer?
A Divide it by 2 **B** Divide it by 4 **C** Multiply it by 6 **D** Divide it by 8
E Multiply it by 8

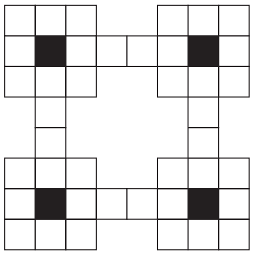
S5. If n is a positive integer, then $2^{n+2003} + 2^{n+2003}$ is equal to
A 2^{n+2004} **B** $2^{2n+4006}$ **C** $4^{2n+4006}$ **D** $4^{2n+2003}$ **E** 4^{n+2003}

S6. For which of the following settings does a triangle ABC exist?
A $AB = 11\,\text{cm}$, $BC = 19\,\text{cm}$, $CA = 7\,\text{cm}$
B $AB = 11\,\text{cm}$, $BC = 7\,\text{cm}$, $\angle BAC = 60^\circ$
C $AB = 11\,\text{cm}$, $CA = 7\,\text{cm}$, $\angle CBA = 128^\circ$
D $AB = 11\,\text{cm}$, $\angle BAC = 60^\circ$, $\angle CBA = 128^\circ$
E For none of them

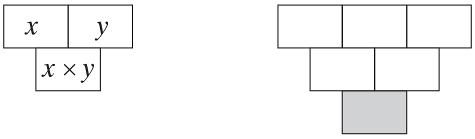
S7. The average number of students accepted by a school in the four years 1998–2001 was 325 students per year. The average number of students accepted by the school in the five years 1998–2002 is 20% higher. How many students did this school accept in 2002?
A 650 **B** 600 **C** 455 **D** 390 **E** 345

S8. Find all values of the parameter m for which the curves $x^2 + y^2 = 1$ and $y = x^2 + m$ have exactly one common point.
A $-\frac{5}{4}; -1; 1$ **B** $-\frac{5}{4}; 1$ **C** $-1; 1$ **D** $-\frac{5}{4}$ **E** 1

S9. The composite board shown in the picture consists of 44 fields 1×1. How many possibilities are there to cover all 40 white fields with 20 rectangular stones 1×2? (The board cannot be turned. Two possibilities are different if at least one stone lies in another way.)
A 8 **B** 16 **C** 32 **D** 64 **E** 100



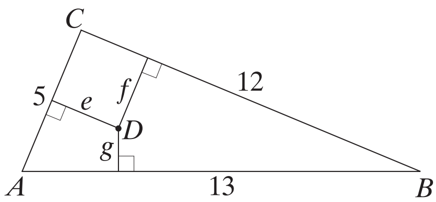
S10. According to the rule given in the left picture below, we construct a numerical triangle with an integer number greater than 1 in each cell. Which of the numbers given in the answers cannot appear in the shaded cell?



A 154 **B** 100 **C** 90 **D** 88 **E** 60

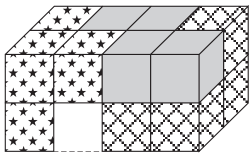
4-POINT QUESTIONS

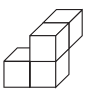
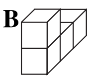
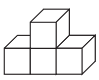
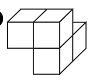
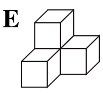
S11. Let ABC be a triangle with area 30. Let D be any point in its interior and let e, f and g denote the distances from D to the sides of the triangle.



What is the value of the expression $5e + 12f + 13g$?
A 120 **B** 90 **C** 60 **D** 30
E Impossible to find the value without knowing the exact location of D

S12. A rectangular parallelepiped was composed of 4 pieces, each consisting of 4 little cubes. Then one piece was removed (see picture). Which one?



A  **B**  **C**  **D**  **E** 

S13. Two white and eight gray seagulls were flying over a river. Suddenly, they all randomly sat down at the bank forming a line. What is the probability that the two white seagulls were sitting side by side?
A $\frac{1}{5}$ **B** $\frac{1}{6}$ **C** $\frac{1}{7}$ **D** $\frac{1}{8}$ **E** $\frac{1}{9}$

S14. The value of

$$\sqrt{1+2000\sqrt{1+2001\sqrt{1+2002\sqrt{1+2003\cdot 2005}}}}$$

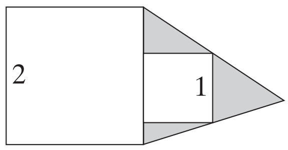
is equal to
A 2000 **B** 2001 **C** 2002 **D** 2003 **E** 2004

S15. Numbers 12, 13 and 15 are the lengths (perhaps not in order) of two sides of an acute-angled triangle and of the height over the third side of this triangle. Find the area of the triangle.
A 168 **B** 80 **C** 84 **D** $6\sqrt{65}$ **E** Impossible to find

S16. The sequence $1^7, 2^7, 3^7, \ldots$ is constructed of the seventh powers of all positive integers. How many terms of this sequence lie between the numbers 5^{21} and 2^{49}?
A 13 **B** 8 **C** 5 **D** 3 **E** 2

S17. We know that $10^n + 1$ is a multiple of 101, and n is a 2-digit number. What is the largest possible value of n?
A 92 **B** 94 **C** 96 **D** 98 **E** 99

S18. The diagram shows two squares: one has a side with a length of 2 and the other (abut on the first square) has a side with a length of 1. What is the area of the shaded zone?
A 1 **B** 2 **C** $2\sqrt{2}$ **D** 4
E It depends on the position of the smaller square

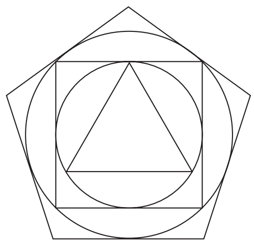


S19. How many of the functions $f(x) = 0,\ f(x) = \frac{1}{2},\ f(x) = 1,\ f(x) = x,\ f(x) = -x$ satisfy the equation $f(x^2 + y^2) = f^2(x) + f^2(y)$?
A 1 **B** 2 **C** 3 **D** 4 **E** 5

S20. If $a^4 + \frac{1}{a^4} = 4$, then $a^6 + \frac{1}{a^6}$ is equal to
A $4\sqrt{6}$ **B** $3\sqrt{6}$ **C** 6 **D** $5\sqrt{6}$ **E** $6\sqrt{6}$

5-POINT QUESTIONS

S21. We first draw an equilateral triangle, then draw the circumcircle of this triangle, then circumscribe a square to this circle. After drawing another circumcircle, we circumscribe a regular pentagon to this circle, and so on. We repeat this construction with new circles and new regular polygons (each with one side more than the preceding one) until we draw a 16-sided regular polygon. How many disjoint regions are there inside the last polygon?
A 232 **B** 240 **C** 248 **D** 264 **E** 272



S22. A point $P(x; y)$ lies on a circle with center $M(2; 2)$ and radius r. We know that $y = r > 2$ and x, y and r are all positive integers. What is the smallest possible value of x?
A 2 **B** 4 **C** 6 **D** 8 **E** 10

S23. The four positive integers A, B, $A - B$, $A + B$ are all prime. Then the sum of them
A is even **B** is a multiple of 3 **C** is a multiple of 5 **D** is a multiple of 7 **E** is prime

S24. A manager in a store has to determine the price of a sweater. Market research gives him the following information: If the price is \$75 then 100 teens will buy the sweaters. The price can be increased or decreased several times by units of \$5. Each time the price is increased by \$5, 20 fewer teens will buy the sweaters. However, each time the price is decreased by \$5, 20 sweaters more will be sold. The sweater costs the company \$30 apiece. What is the sale price that maximizes profits?
A 85 **B** 80 **C** 75 **D** 70 **E** 65

S25. How many distinct pairs $(x; y)$ satisfy the equation $(x + y)^2 = (x + 3)(y - 3)$?
A 0 **B** 1 **C** 2 **D** 3 **E** Infinitely many

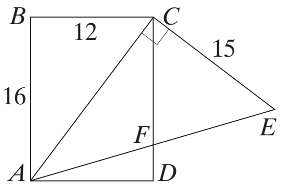
S26. A sequence $a_0, a_1, a_2, \ldots$ is defined in the following way:

$$a_0 = 4, \quad a_1 = 6, \quad a_{n+1} = \frac{a_n}{a_{n-1}} \qquad (n \geqslant 1).$$

Then a_{2003} is equal to

A $\frac{3}{2}$ **B** $\frac{2}{3}$ **C** 4 **D** $\frac{1}{4}$ **E** $\frac{1}{6}$

S27. In the picture $ABCD$ is a rectangle with $AB = 16$, $BC = 12$. Let E be such a point that $AC \perp CE$, $CE = 15$.



If F is the point of intersection of segments AE and CD, then the area of the triangle ACF is equal to
A 75 **B** 80 **C** 96 **D** 72 **E** 48

S28. We can put an arrow on one end of the edge of a cube, defining a vector, and put an arrow on the other end of the edge, defining the opposite vector. We put an arrow on each edge and then add up all 12 vectors obtained. How many different values of sum of vectors can we obtain in this way?
A 25 **B** 27 **C** 64 **D** 100 **E** 125

S29. We are given the 6 vertices of a regular hexagon and all line segments joining any two of these points. We call two such segments *strangers* if they have no common point (including end points). How many pairs of strangers are there?
A 26 **B** 28 **C** 30 **D** 34 **E** 36

S30. Let f be a polynomial such that $f(x^2 + 1) = x^4 + 4x^2$. Determine $f(x^2 - 1)$.
A $x^4 - 4x^2$ **B** x^4 **C** $x^4 + 4x^2 - 4$ **D** $x^4 - 4$ **E** Another answer