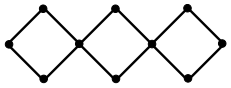
22. What is the set of all values of the parameter a for which the number of solutions of the equation $2 - |x| = ax$ is equal to two?
A) $(-\infty; -1]$ **B)** $[1; +\infty)$ **C)** $(-1; 1)$ **D)** $\{0\}$ **E)** $\{-1, 1\}$

23. The integers from 1 to 99 are written in ascending order without gaps. The sequence of digits is then divided into triplets of digits:(123)(456)(789)(101)(112)...(979)(899). Which of the following is not one of the triplets?
A) (222) **B)** (444) **C)** (464) **D)** (646) **E)** (888)

24. The vertices of the network shown are labelled with the numbers from 1 to 10. The sum S of the four labels on each square is the same. What is the least possible value of S?



A) 18 **B)** 19 **C)** 20 **D)** 21 **E)** 22

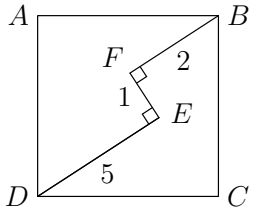
25. For how many integers n is $|n^2 - 2n - 3|$ a prime number?
A) 1 **B)** 2 **C)** 3 **D)** 4 **E)** Infinitely many

26. Four straight lines pass through the origin of the coordinate system. They intersect the parabola $y = x^2 - 2$ at eight distinct points. What can be the product of the x-coordinates of these eight points?
A) Only 16 **B)** Only -16 **C)** Only 8 **D)** Only -8
E) There are several possible products

27. The sequence a_1, a_2, a_3, ... starts with $a_1 = 16$. For $n \geq 1$, the number a_{n+1} is obtained by adding 1 to the sum of the digits of a_n and then squaring the result. Thus $a_2 = (1 + 6 + 1)^2 = 64$. Determine a_{2019}.
A) 16 **B)** 25 **C)** 64 **D)** 100 **E)** 121

28. How many planes pass through at least three vertices of a given cube?
A) 6 **B)** 8 **C)** 12 **D)** 16 **E)** 20

29. A path $DEFB$ with $DE \perp EF$ and $EF \perp FB$ lies inside the square $ABCD$. Given that $DE = 5$, $EF = 1$ and $FB = 2$, what is the length of the side of the square?
A) 5 **B)** $\frac{7\sqrt{2}}{2}$ **C)** $\frac{11}{2}$ **D)** $5\sqrt{2}$ **E)** $3\sqrt{2}$



30. Three different numbers are chosen at random from the set $\{1, 2, 3, \ldots, 10\}$. What is the probability that one of them is the average of the other two?
A) $\frac{1}{10}$ **B)** $\frac{1}{6}$ **C)** $\frac{1}{4}$ **D)** $\frac{1}{3}$ **E)** $\frac{1}{2}$

Lietuvos Respublikos švietimo ir mokslo ministerija
Kengūros konkurso organizavimo komitetas
Vilniaus universitetas
Lietuvos matematikų draugija

KANGAROO 2019

S

Time allowed: 75 minutes
Calculators are not permitted

Student
11–12 grades

Questions for 3 points

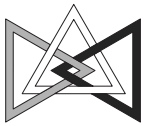
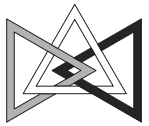
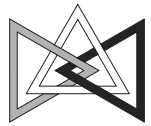
1. The flag of Kangoraland is a rectangle which is divided into three smaller equal rectangles as shown. What is the ratio of the side lengths of the flag?

A) 2 : 1 **B)** 3 : 2 **C)** 5 : 3 **D)** 8 : 5 **E)** 9 : 4

2. The numbers 1, 2, 3 and 4 are each written in different cells of the 2×2 table. After that, the sum of the numbers in each row and column is calculated. Two of these sums are 4 and 5. What are the other two sums?

A) 6 and 6 **B)** 3 and 5 **C)** 4 and 5 **D)** 4 and 6 **E)** 5 and 6

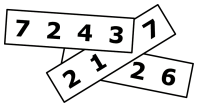
3. Three triangles are linked as shown. Which of the following pictures shows these three triangles linked in the same way?

A)  **B)**  **C)**  **D)**  **E)** 

4. What is the first (leftmost) digit of the smallest positive integer whose digits add up to 25?
A) 1 **B)** 3 **C)** 5 **D)** 7 **E)** 9

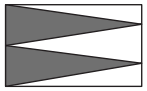
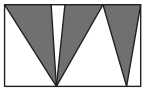
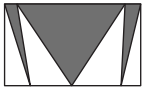
5. A pyramid has 23 triangular faces. How many edges does this pyramid have?
A) 23 **B)** 24 **C)** 46 **D)** 48 **E)** 69

6. Three 4-digit numbers are written on three pieces of paper as shown. The sum of the three numbers is 11126. Three of the digits are covered. What are the covered digits?

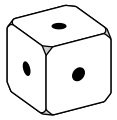
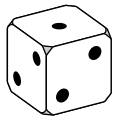


A) 1, 4, 7 **B)** 1, 5, 7 **C)** 3, 3, 3 **D)** 4, 5, 6 **E)** 4, 5, 7

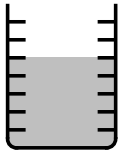
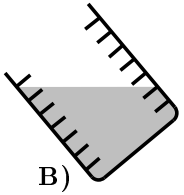
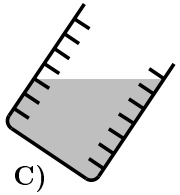
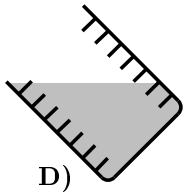
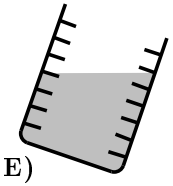
7. A rectangle has been shaded in five different ways as shown. In which diagram does the shaded part have the largest area?

A)  **B)**  **C)**  **D)**  **E)** 

8. Each of the faces of a die is marked with either 1, 2 or 3 dots so that the probability of rolling a 1 is $\frac{1}{2}$, the probability of rolling a 2 is $\frac{1}{3}$ and the probability of rolling a 3 is $\frac{1}{6}$. Which of the following cannot be a view of this die?

A)  **B)**  **C)**  **D)**  **E)** 

9. Five identical glasses are filled with water. Four of them contain the same amount of water. Which one contains a different amount?

A)  **B)**  **C)**  **D)**  **E)** 

10. How many of the numbers from 2^{10} to 2^{13}, inclusive, are divisible by 2^{10}?
A) 2 **B)** 4 **C)** 6 **D)** 8 **E)** 16

Questions for 4 points

11. Michael invented a new "$\diamondsuit$" operation for real numbers, defined as $x \diamondsuit y = y - x$. If a, b, and c satisfy $(a \diamondsuit b) \diamondsuit c = a \diamondsuit (b \diamondsuit c)$, which of the following statements is necessarily true?
A) $a = b$ **B)** $b = c$ **C)** $a = c$ **D)** $a = 0$ **E)** $c = 0$

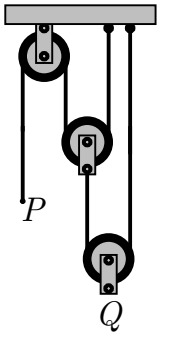
12. The numbers a, b, c, d are distinct positive integers chosen from 1 to 10. What is the least possible value $\frac{a}{b} + \frac{c}{d}$ could have?

A) $\frac{2}{10}$ **B)** $\frac{3}{19}$ **C)** $\frac{14}{45}$ **D)** $\frac{29}{90}$ **E)** $\frac{25}{72}$

13. Three kangaroos, Alex, Bob and Carl, go for a walk every day. If Alex doesn't wear a hat, then Bob wears a hat. If Bob doesn't wear a hat, then Carl wears a hat. Today Carl is not wearing a hat. Who is certainly wearing a hat today?
A) Only Alex and Bob **B)** Only Alex **C)** Alex, Bob and Carl
D) Neither Alex nor Bob **E)** Only Bob

14. Which is the highest power of 3 dividing the number $7! + 8! + 9!$?
A) 3^2 **B)** 3^4 **C)** 3^5 **D)** 3^6 **E)** 3^7

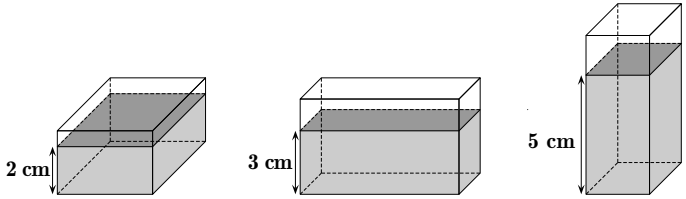
15. The system shown consists of three pulleys with vertical sections of rope between them. The end P is moved down 24 centimetres. How many centimetres does point Q move up?
A) 24 **B)** 12 **C)** 8 **D)** 6 **E)** $\frac{24}{5}$



16. This year, the number of boys in my class has increased by 20% and the number of girls has decreased by 20%. We now have one student more than before. Which of the following could be the number of students in my class now?
A) 22 **B)** 26 **C)** 29 **D)** 31 **E)** 34

17. What is the integer part of $\sqrt{20 + \sqrt{20 + \sqrt{20 + \sqrt{20 + \sqrt{20}}}}}$?
A) 4 **B)** 5 **C)** 6 **D)** 20 **E)** 25

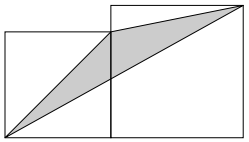
18. A container in the shape of a rectangular box is partially filled with 120 cm^3 of water. The depth of the water is either 2 cm or 3 cm or 5 cm, depending on which side of the box is on the ground, as shown. What is the volume of the container?



A) 160 cm^3 **B)** 180 cm^3 **C)** 200 cm^3 **D)** 220 cm^3 **E)** 240 cm^3

19. A positive integer n is called *good* if its largest divisor (excluding n) is equal to $n - 6$. How many *good* positive integers are there?
A) 1 **B)** 2 **C)** 4 **D)** 6 **E)** None of the previous

20. Two adjacent squares with side lengths a and b ($a < b$) are shown. What is the area of the shaded triangle?
A) $\frac{1}{2}a^2$ **B)** $\sqrt{ab}$ **C)** $\frac{1}{2}b^2$ **D)** $\frac{1}{4}(a^2 + b^2)$ **E)** $\frac{1}{2}(a^2 + b^2)$



Questions for 5 points

21. Let a be the sum of all positive divisors of 1024 and b the product of all positive divisors of 1024. Then
A) $(a-1)^5 = b$ **B)** $(a+1)^5 = b$ **C)** $a^5 = b$ **D)** $a^5 - 1 = b$ **E)** $a^5 + 1 = b$