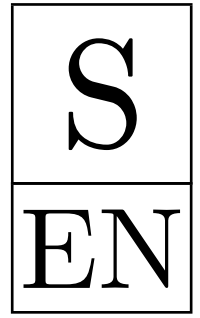# KANGAROO 2021

S

EN

Time allowed: 75 minutes
Calculators are not permitted
*The participants solve problems independently*

Student
11–12 grades

Questions for 3 points

1. Each year, the third Thursday in March is named Kangaroo Day. The dates of Kangaroo Day for the next few years are shown below, with one error. Which date is wrong?
A) 2022 17 March   **B)** 2023 16 March   **C)** 2024 14 March   **D)** 2025 20 March   **E)** 2026 19 March

2. Paula's weather app shows a diagram of the predicted weather and maximum temperatures for the next seven days, as shown. Which of the following represents the corresponding graph of maximum temperatures?
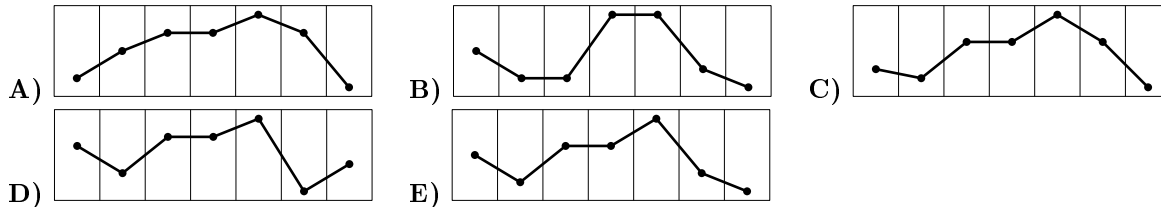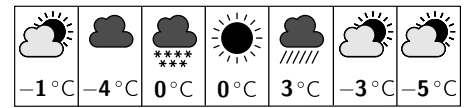
**3.** A cube with edge 1 is cut into two identical cuboids. What is the surface area of one of these cuboids?
A) $\frac{3}{2}$ **B)** 2   **C)** 3   **D)** 4   **E)** 5

4. How many integers are in the interval $(20 - \sqrt{21}; 20 + \sqrt{21})$?
A) 9   **B)** 10   **C)** 11   **D)** 12   **E)** 13

5. A large square is divided into smaller squares, as shown. A shaded circle is inscribed inside each of the smaller squares. What proportion of the area of the large square is shaded?
A) $\frac{6\pi}{25}$ **B)** $\frac{\pi}{5}$ **C)** $\frac{3\pi}{10}$ **D)** $\frac{\pi}{4}$ **E)** $\frac{5\pi}{16}$
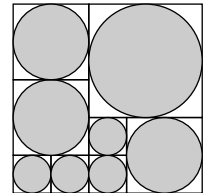

6. Let $x = \frac{\pi}{4}$. Which of the following numbers is the largest?
A) x^4 **B)** x^2 **C)** x **D)** $\sqrt{x}$ **E)** $\sqrt[4]{x}$

7. A rectangular sheet of paper has length x and width y, where $x > y$. The rectangle may be folded to form the curved surface of a circular cylinder in two different ways. What is the ratio of the volume of the longer cylinder to the volume of the shorter cylinder?
A) $y^2 : x^2$ **B)** $y : x$ **C)** 1:1   **D)** $x : y$ **E)** $x^2 : y^2$

8. After the storm last night, the flagpole on our school building is leaning over. Looking from northwest, its tip is to the right of its bottom point. Looking from the east, its tip is also to the right of its bottom point. In which direction could the flagpole be leaning over?
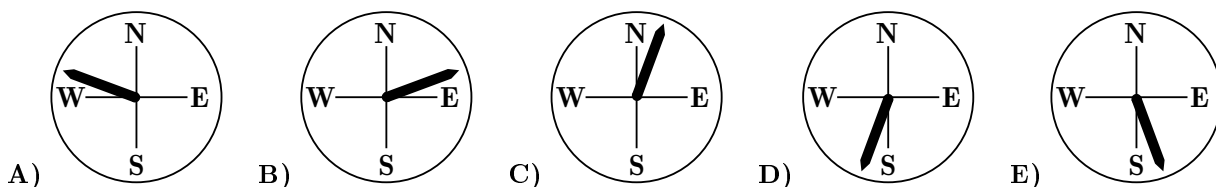
9. What is the area of the triangle whose vertices are at $(p; q)$, $(3p; q)$ and $(2p; 3q)$, where $p, q > 0$?
A) $\frac{pq}{2}$ **B)** pq **C)** $2pq$ **D)** $3pq$ **E)** $4pq$

10. How many 3-digit positive integers formed using only the digits 1, 3 and 5 are divisible by 3? You may use digits more than once.
A) 3   **B)** 6   **C)** 9   **D)** 18   **E)** 27

Questions for 4 points

11. What proportion of all the positive divisors of 7! is odd?
A) $\frac{1}{2}$ **B)** $\frac{1}{3}$ **C)** $\frac{1}{4}$ **D)** $\frac{1}{5}$ **E)** $\frac{1}{6}$

12. If $A = (0; 1) \cup (2; 3)$ and $B = (1; 2) \cup (3; 4)$, what is the set of all numbers of the form $a + b$ with $a \in A$ and $b \in B$?
**A)** $(1; 3) \cup (3; 5) \cup (5; 7)$ **B)** $(1; 5) \cup (5; 7)$ **C)** $(1; 3) \cup (3; 7)$ **D)** $(1; 7)$ **E)** None of the previous

13. How many three-digit positive integers have the property that when their digits are written in reverse order, the result is a number which is 99 more than the original number?
**A)** 8   **B)** 64   **C)** 72   **D)** 80   **E)** 81

14. The parabola in the figure has an equation $y = ax^2 + bx + c$ for some distinct real numbers a, b and c. Which of the following equations could be an equation of the line in the figure?
A) $y = bx + c$ **B)** $y = cx + b$ **C)** $y = ax + b$ **D)** $y = ax + c$ **E)** $y = cx + a$
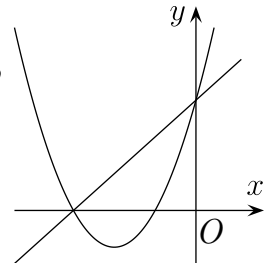
15. A large triangle is divided into smaller triangles as shown. The number inside each small triangle indicates its perimeter. What is the perimeter of the large triangle?
A) 31   **B)** 34   **C)** 41   **D)** 62   **E)** None of the previous
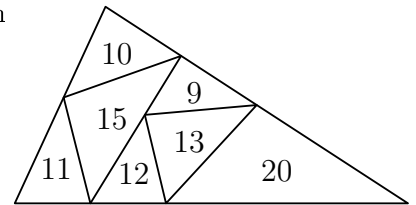
16. For a number N, we denote by $p(N)$ the product of the digits of N. For example, $p(23) = 2 \cdot 3 = 6$. What is the value of the sum $p(10) + p(11) + p(12) + \ldots + p(99) + p(100)$?
A) 2025   **B)** 4500   **C)** 5005   **D)** 5050   **E)** None of the previous

17. In the 5×5 square shown the sum of the numbers in each row and in each column is the same. There is a number in every cell, but some of the numbers are not shown. What is the number in the cell marked with a question mark?
A) 8   **B)** 10   **C)** 12   **D)** 18   **E)** 23

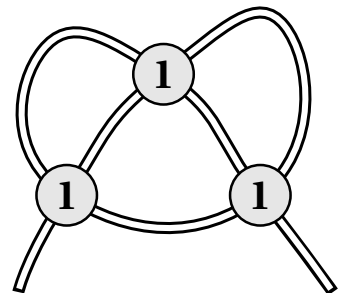|	16		22	
20		21		2
	25		1	
24		5		6
	4		?	

18. A piece of string is lying on the table. It is partially covered by three coins as seen in the figure. Under each coin the string is equally likely to pass over itself like this: or like this: . What is the probability that the string is knotted after its ends are pulled?
A) $\frac{1}{2}$ **B)** $\frac{1}{4}$ **C)** $\frac{1}{8}$ **D)** $\frac{3}{4}$ **E)** $\frac{3}{8}$

19. The first 1000 positive integers are written in a row in some order and all sums of any three adjacent numbers are calculated. What is the greatest number of odd sums that can be obtained?
**A)** 997   **B)** 996   **C)** 995   **D)** 994   **E)** 993

20. The diagram shows three squares, $ABCD$, $DEFG$ and $FHKL$. They are placed together, edge to edge. Points B, E and K lie on the same straight line. The area of $ABCD$ is 36 and the area of $DEFG$ is 16. What is the area of triangle BGK?
**A)** $14\frac{2}{3}$ **B)** $15\frac{1}{3}$ **C)** 16   **D)** $17\frac{2}{3}$ **E)** 18
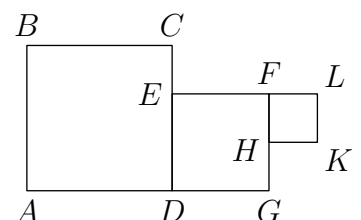
21. The figure shows the graph of a function $f : [-5; 5] \to \mathbb{R}$. How many distinct solutions does the equation $f(f(x)) = 0$ have?
**A)** 2   **B)** 4   **C)** 6   **D)** 7   **E)** 8
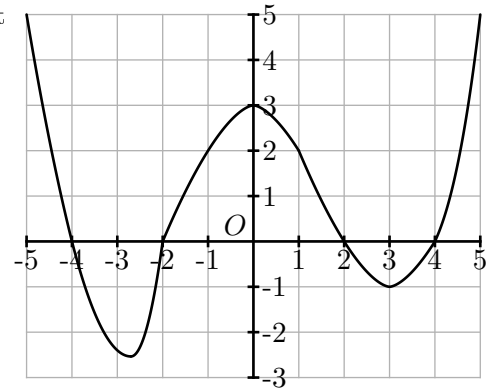
22. The numbers 1, 2, 7, 9, 10, 15 and 19 are written down on a blackboard. Two players alternately delete one number each until only one number remains on the blackboard. The sum of the numbers deleted by one of the players is twice the sum of the numbers deleted by the other player. What is the number that remains?
A) 2   **B)** 7   **C)** 9   **D)** 10   **E)** 19

23. The function $f(x)$ defined on positive integers is such that $f(x + y) = f(x) \cdot f(y)$ and $f(1) = 2$. What is the value of $\dfrac{f(2)}{f(1)} + \dfrac{f(3)}{f(2)} + \cdots + \dfrac{f(2021)}{f(2020)}$?
A) 0   **B)** $\dfrac{1}{2}$ **C)** 2   **D)** 2020   **E)** None of the previous

24. An ant climbs from C to A on path CA and descends from A to B on the stairs, as shown in the diagram. What is the ratio of the lengths of the descending and ascending paths?
**A)** 1   **B)** 2   **C)** 3   **D)** $\sqrt{2}$ **E)** $\sqrt{3}$
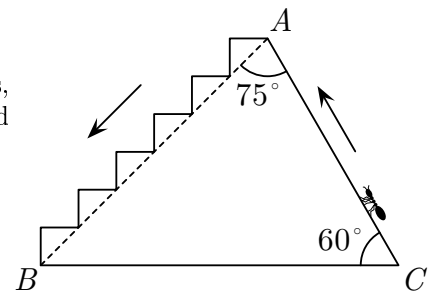
25. The solid shown in the diagram has 12 regular pentagonal faces, the other faces being either equilateral triangles or squares. Each pentagonal face is surrounded by 5 square faces and each triangular face is surrounded by 3 square faces. John writes 1 on each triangular face, 5 on each pentagonal face and -1 on each square. What is the total of the numbers written on the solid?
**A)** 20   **B)** 50   **C)** 60   **D)** 80   **E)** 120
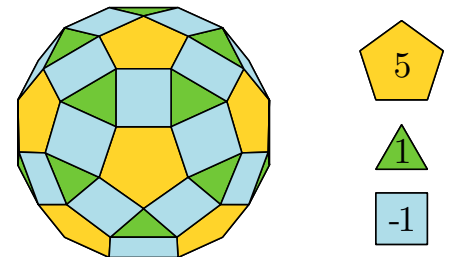
26. Each of the numbers a and b is a square of an integer. The difference $a - b$ is a prime number. Which of the following could be neither a nor b?
A) 144   **B)** 400   **C)** 625   **D)** 729   **E)** 2500

27. A triangle ABC is divided into four parts by two straight lines. The areas of the smaller triangles are 1, 3 and 3, as shown. What is the area of the original triangle?
A) 12   **B)** 12.5   **C)** 13   **D)** 13.5   **E)** 14
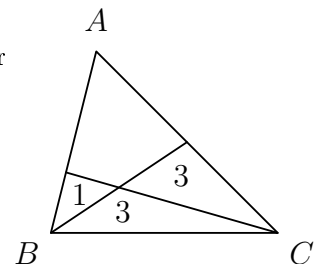
28. Five kangaroos named A, B, C, D and E have one child each, named a, b, c, d and e. The picture on the left shows exactly two of the children standing next to their mothers. The picture on the right shows exactly three of the children standing next to their mothers. Whose child is a?
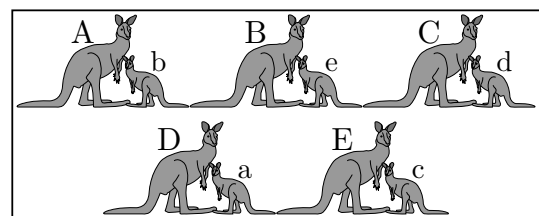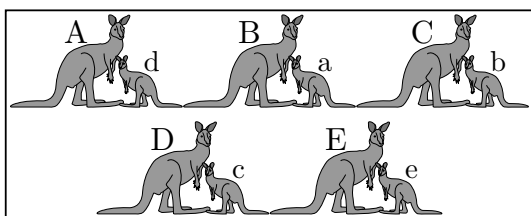
A) A   **B)** B   **C)** C   **D)** D   **E)** E

29. Let $M(k)$ be the maximum value of $|4x^2 - 4x + k|$ for $x \in [-1; 1]$, where k can be any real number. What is the minimum possible value of $M(k)$?

A) 4   **B)** $4\frac{1}{2}$ **C)** 5   **D)** $5\frac{1}{2}$ **E)** 8

30. Two plane mirrors OP and OQ are inclined at an acute angle. A ray of light XY parallel to QO strikes mirror OP at Y. The ray is reflected and hits mirror OQ, is reflected again and hits mirror OP and is reflected for a third time and strikes mirror OQ at right angle at R. The distance OR is $5\,\text{cm}$. The ray XY is $d\,\text{cm}$ from the mirror OQ. What is the value of d?

A) 4   **B)** $4\frac{1}{2}$ **C)** 5   **D)** $5\frac{1}{2}$ **E)** 6