

24. How many solutions (x, y) , where x and y are real numbers, does the equation $x^2 + y^2 = |x| + |y|$ have?

- A) 1 B) 5 C) 8 D) 9 E) Infinitely many

25. Let \mathbb{N}_0 be the set of non-negative integers. Let $f : \mathbb{N}_0 \rightarrow \mathbb{N}_0$ be the function defined by $f(2n) = f(2n + 1) = n$ for all non-negative integers n . For every k , $f^k(n)$ denotes the number represented by the expression $f(f(\dots f(n)\dots))$, where the symbol f appears k times. The number of solutions of the equation $f^{2013}(n) = 1$ is:

- A) 0 B) 4026 C) 2^{2012} D) 2^{2013} E) Infinite

26. There are some straight lines drawn on the plane. Line a intersects exactly three other lines and line b intersects exactly four other lines. Line c intersects exactly n other lines, with $n \neq 3, 4$. Determine the number of lines drawn on the plane.

- A) 4 B) 5 C) 6 D) 7 E) Another number

27. The sum of the first n positive integers is a three-digit number in which all of the digits are the same. What is the sum of the digits of n ?

- A) 6 B) 9 C) 12 D) 15 E) 18

28. On the island of Knights and Knaves there live only two types of people: Knights (who always speak the truth) and Knaves (who always lie). I met two men who lived there and asked the taller man if they were both Knights. He replied, but I could not figure out what they were, so I asked the shorter man if the taller was a Knight. He replied, and after that I knew which type they were. Were the men Knights or Knaves?

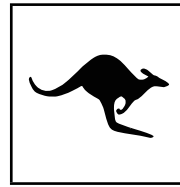
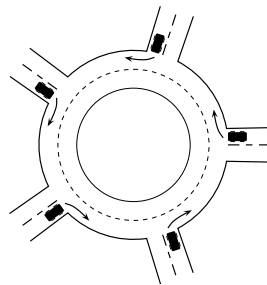
- A) They were both Knights
B) They were both Knaves
C) The taller was a Knight and the shorter was a Knave
D) The taller was a Knave and the shorter was a Knight
E) Not enough information is given

29. Iulian has written an algorithm in order to create a sequence of numbers as $a_1 = 1$, $a_{m+n} = a_m + a_n + mn$, where m and n are positive integers. Find the value of a_{100} .

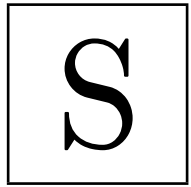
- A) 100 B) 1000 C) 2012 D) 4950 E) 5050

30. The roundabout shown in the picture is entered by 5 cars at the same time, each one from a different direction. Each of the cars drives less than one round and no two cars leave the roundabout in the same direction. How many different combinations are there for the cars leaving the roundabout?

- A) 24 B) 44 C) 60 D) 81 E) 120



KANGAROO 2013



Time allowed: 75 min
Calculators are not permitted

Student
11–12 grades

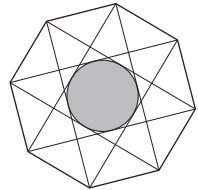
Questions for 3 points

1. Which of the following numbers is the largest?

- A) 2013 B) 2^{0+13} C) 20^{13} D) 201^3 E) $20 \cdot 13$

2. The regular octagon of the figure measures 10 on each side. Which is the measure of the radius of the circle inscribed in the smallest octagon formed by the diagonals?

- A) 10 B) 7,5 C) 5 D) 2,5 E) 2



3. A prism has 2013 faces in total. How many edges has the prism?

- A) 2011 B) 2013 C) 4022 D) 4024 E) 6033

4. The cube root of 3^3 is equal to:

- A) 3^3 B) 3^{3^3-1} C) 3^{2^3} D) 3^{3^2} E) $(\sqrt{3})^3$

5. The year 2013 has the property that its number is made up of the consecutive digits 0, 1, 2 and 3. How many years have passed since the last time a year was made up of some four consecutive digits?

- A) 467 B) 527 C) 581 D) 693 E) 990

6. Let f be a linear function for which $f(2013) - f(2001) = 100$. What is $f(2031) - f(2013)$?

- A) 75 B) 100 C) 120 D) 150 E) 180

7. Given that $2 < x < 3$ how many of the statements

$$4 < x^2 < 9, \quad 4 < 2x < 9, \quad 6 < 3x < 9, \quad 0 < x^2 - 2x < 3$$

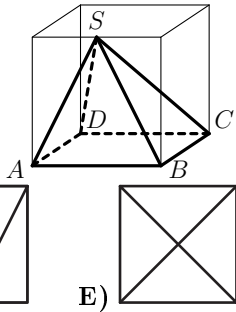
are necessarily true?

- A) 0 B) 1 C) 2 D) 3 E) 4

8. Six superheroes capture 20 villains. The first superhero captures one villain, the second captures two villains and the third captures three villains. The fourth superhero captures more villains than any of the other five. What is the smallest number of villains the fourth superhero must have captured?

- A) 7 B) 6 C) 5 D) 4 E) 3

9. In the transparent cube below you see a solid not transparent pyramid $ABCD S$ with base $ABCD$, whose vertex S lies exactly in the middle of an edge of the cube. You look at this pyramid from above, from below, from behind, from ahead, from the right and from the left. Which view does not arise?

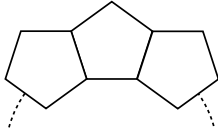


- A) B) C) D) E)

10. When a certain solid substance melts, its volume increases by $\frac{1}{12}$. By how much does its volume decrease when it solidifies again?
 A) $\frac{1}{10}$ B) $\frac{1}{11}$ C) $\frac{1}{12}$ D) $\frac{1}{13}$ E) $\frac{1}{14}$

Questions for 4 points

11. Radu has identical plastic pieces in the shape of a regular pentagon. He glues them edge to edge to complete a circle - as shown in the picture. How many pieces are there in this circle?



- A) 8 B) 9 C) 10 D) 12 E) 15

12. How many positive integers n exist such that both $\frac{n}{3}$ and $3n$ are three-digit integers?
 A) 12 B) 33 C) 34 D) 100 E) 300

13. A circular carpet is placed on a floor of square tiles. All the tiles which have more than one point in common with the carpet are marked grey. Which of the following is an impossible outcome?

- A) B) C) D) E)

14. Consider the following proposition about a function f which is defined on the set of integers and takes integer values: "For any even x , $f(x)$ is even." If this proposition is false then it follows that:

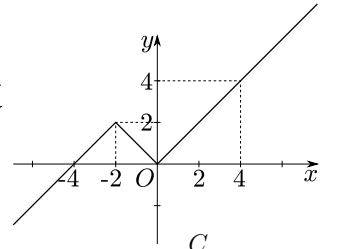
- A) For any even x , $f(x)$ is odd
 B) For any odd x , $f(x)$ is even
 C) For any odd x , $f(x)$ is odd
 D) There exists an even number x such that $f(x)$ is odd
 E) There exists an odd number x such that $f(x)$ is odd

15. Given a function $W(x) = (a-x)(b-x)^2$, where $a < b$. Its graph is in one of the following figures. In which one?

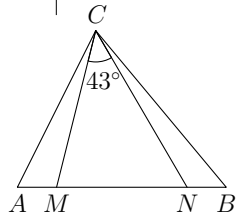
- A) B) C) D) E)

16. Consider a rectangle, one of whose sides has length 5. The rectangle can be cut into a square and a rectangle, one of which has the area 4. How many such rectangles exist?
 A) 1 B) 2 C) 3 D) 4 E) 5

17. Vlad has drawn the graph of a function $f : \mathbb{R} \rightarrow \mathbb{R}$, composed of two rays and a line segment (see figure). How many solutions does the equation $f(f(f(x))) = 0$ have?
 A) 4 B) 3 C) 2 D) 1 E) 0



18. In the triangle ABC the points M and N on the side AB are such that $AN = AC$ and $BM = BC$. Find $\angle ACB$ if $\angle MCN = 43^\circ$.
 A) 86° B) 89° C) 90° D) 92° E) 94°



19. How many pairs (x, y) of positive integers satisfy the equation $x^2 y^3 = 6^{12}$?
 A) 6 B) 8 C) 10 D) 12 E) Another number

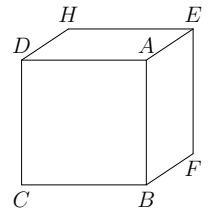
20. A box contains 900 cards numbered from 100 to 999. Any two cards have different numbers. François picks some cards and determines the sum of the digits on each of them. At least how many cards must he pick in order to be certain to have three cards with the same sum?
 A) 51 B) 52 C) 53 D) 54 E) 55

Questions for 5 points

21. How many pairs (x, y) of integers with $x \leq y$ exist such that their product equals 5 times their sum?
 A) 4 B) 5 C) 6 D) 7 E) 8

22. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by the following properties: f is periodic with period 5 and the restriction of f to $[-2, 3)$ is $x \mapsto f(x) = x^2$. What is $f(2013)$?
 A) 0 B) 1 C) 2 D) 4 E) 9

23. The solid cube in the figure is cut by a plane passing through the three neighbouring vertices D , E and B of A . Similarly, the cube is cut by planes passing through the three neighbouring vertices of all other seven corners. What will the piece containing the center of the cube look like?



- A) B) C) D) E)