

Lietuvos Respublikos švietimo ir mokslo ministerija  
Kengūros konkurso organizavimo komitetas  
VU Matematikos ir informatikos fakultetas  
VU Matematikos ir informatikos institutas  
Leidykla TEV

## KANGAROO 2012



**Nipper**  
1 and 2 grades

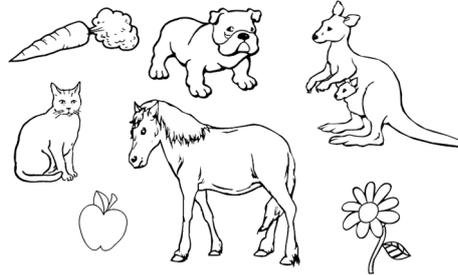
*Time allowed: 50 min*

*Calculators are not permitted*

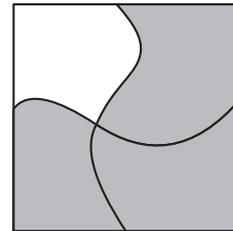
### 3-point questions

1. How many animals are in the picture?

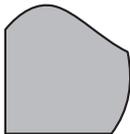
- A) 3 B) 4 C) 5 D) 6 E) 7



2. Which piece fits in the empty place?



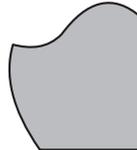
A)



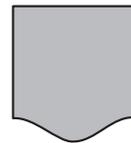
B)



C)



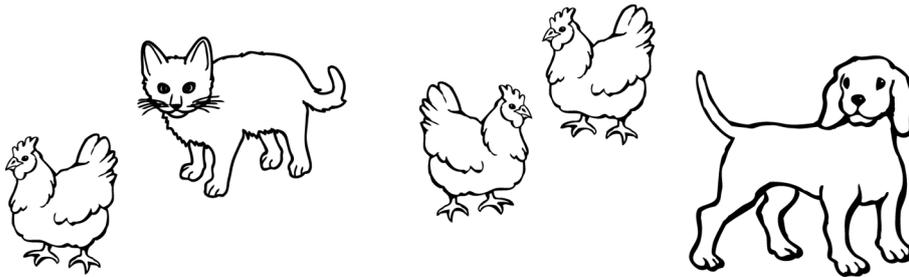
D)



E)



3. How many legs do they have altogether?

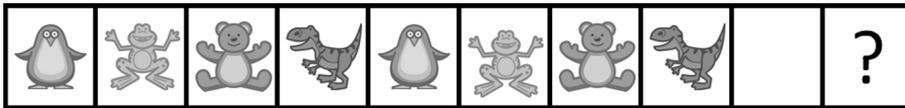


- A) 5   B) 10   C) 12   D) 14   E) 20

4. Helena has written down the word MATHEMATICIAN three times. How many times did she write the letter A?

- A) 3   B) 12   C) 6   D) 9   E) 10

5. Luke repeats the same four stickers on a strip.



Which is the tenth sticker put by Luke?

- A)   B)   C)   D)   E)

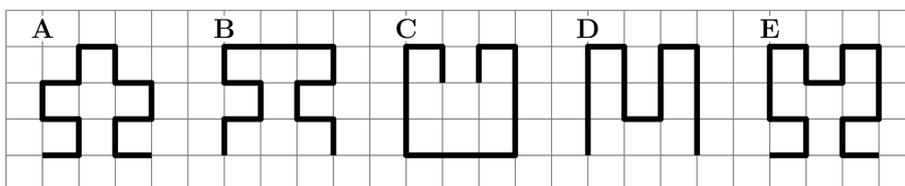
6. What number is covered by the flower?

- A) 1   B) 2   C) 3   D) 4   E) 5

$$\begin{aligned} \bigcirc + \triangle &= 3 \\ \triangle + \triangle &= 4 \\ \triangle + \square &= 5 \\ \bigcirc + \square &= \text{flower} \end{aligned}$$

**4-point questions**

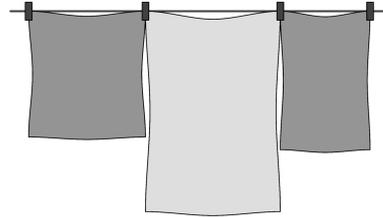
7. Which of the following lines is the longest?



- A) A   B) B   C) C   D) D   E) E

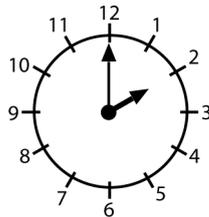
8. Grandmother made 11 cookies. She decorated 6 cookies with raisins and then 7 cookies with nuts. At least how many cookies were decorated with both raisins and nuts?  
 A) 2   B) 3   C) 5   D) 7   E) 11
9. 13 children are playing hide and seek. One of them is the “seeker”. After a while 9 children have been found. How many children are still hiding?  
 A) 3   B) 4   C) 5   D) 9   E) 22

10. Father hangs the laundry outside on a clothesline. He wants to use as few pegs as possible. For 3 towels he needs 4 pegs, as shown. How many pegs does he need for 9 towels?  
 A) 9   B) 10   C) 12   D) 8   E) 18



11. Today Betty added her age and her sister’s age and obtained 10 as the sum. What will the sum of their ages be after one year?  
 A) 5   B) 10   C) 11   D) 12   E) 20

12. The clock shows the time when Stephen leaves his school. School lunch starts 3 hours before school ends.

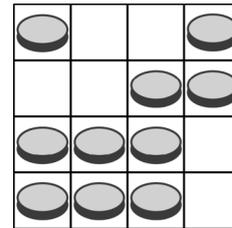


- At what time does lunch start?  
 A) 17   B) 16   C) 12   D) 11   E) 10

**5-point questions**

13. A dragon has 3 heads. Every time a hero cuts off 1 head, 3 new heads emerge. The hero cuts 1 head off and then he cuts 1 off head again. How many heads does the dragon have now?  
 A) 4   B) 5   C) 6   D) 7   E) 8

14. There are coins on the board. We want to have 2 coins in each column and 2 coins in each row. How many coins need to be removed?

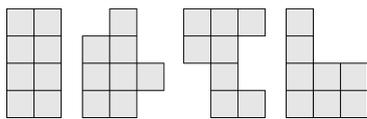


A) 0 B) 1 C) 2 D) 3 E) 4

15. In a box there are three boxes, each one of which contains three smaller boxes. How many boxes are there in total?

A) 9 B) 10 C) 12 D) 13 E) 15

16. Ann has a lot of these tiles: . How many of the following shapes can Ann make by gluing together two of these tiles?

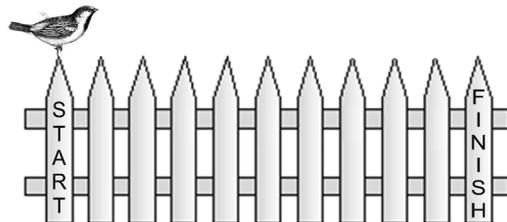


A) 0 B) 1 C) 2 D) 3 E) 4

17. Winnie the Pooh bought 4 apple pies and Eeyore bought 6 cheese cakes. They paid the same and together they paid 24 euros. How many euros does 1 cheese cake cost?

A) 2 B) 4 C) 6 D) 10 E) 12

18. Sparrow Jack jumps on a fence from one post to another. Each jump takes him 1 second. He makes 4 jumps ahead, then 1 jump back and again 4 jumps ahead and 1 back etc. In how many seconds does Jack get from START to FINISH?



A) 10 B) 11 C) 12 D) 13 E) 14

# KANGAROO 2012



**Minor**  
**3 and 4 grades**

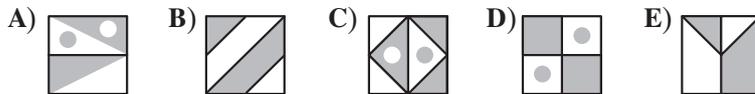
*Time allowed: 75 min*  
*Calculators are not permitted*

### 3-point questions

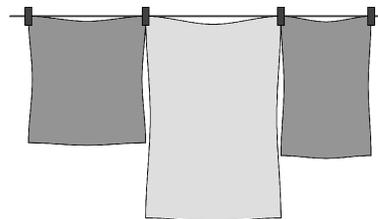
1. Basil wants to write the word MATHEMATICS on a sheet of paper. He wants different letters to be coloured differently, and the same letters to be coloured identically. How many colours will he need?

A) 5 B) 6 C) 7 D) 8 E) 9

2. In four of the five pictures the white area is equal to the grey area. In which picture are the white area and the grey area different?



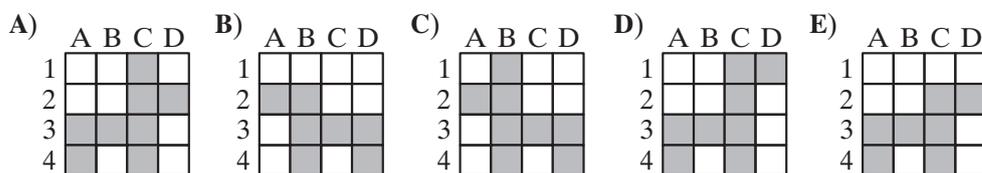
3. Father hangs the laundry outside on a clothesline. He wants to use as few pegs as possible. For 3 towels he needs 4 pegs, as shown. How many pegs does he need for 9 towels?



A) 9 B) 10 C) 12 D) 8 E) 18

4. Iljo colours the squares A2, B1, B2, B3, B4, C3, D3 and D4. Which colouring does he get?

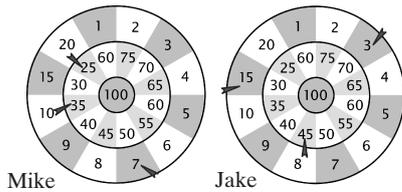
	A	B	C	D
1				
2				
3				
4				



5. 13 children are playing hide and seek. One of them is the “seeker” and the others hide. After a while 9 children have been found. How many children are still hiding?

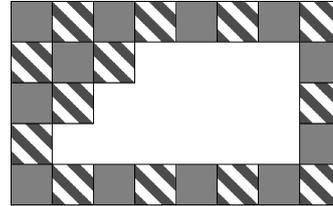
A) 3 B) 4 C) 5 D) 9 E) 22

6. Mike and Jake were playing darts. Each one threw three darts (see the picture). Who won and how many more points did he score?



- A) Mike, he scored 3 points more  
 B) Jake, he scored 4 points more  
 C) Mike, he scored 2 points more  
 D) Jake, he scored 2 points more  
 E) Mike, he scored 4 points more

7. A regular rectangular pattern on a wall was created with 2 kinds of tiles: grey and striped. Some tiles have fallen off the wall (see the picture). How many grey tiles have fallen off?



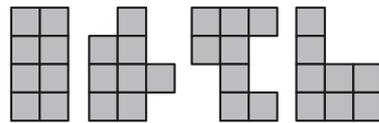
- A) 9 B) 8 C) 7 D) 6 E) 5

8. The year 2012 is a leap year, that means there are 29 days in February. Today, on the 15th March 2012, the ducklings of my grandfather are 20 days old. When did they hatch from their eggs?

- A) On 19th of February B) On 21th of February C) On 23rd of February  
 D) On 24th of February E) On 26th of February

**4-point questions**

9. Ann has a lot of these tiles: . How many of the following shapes can Ann make by glueing together two of these tiles?



- A) 0 B) 1 C) 2 D) 3 E) 4

10. Three balloons cost 12 cents more than one balloon. How much does one balloon cost?

- A) 4 B) 6 C) 8 D) 10 E) 12

11. Grandmother made 20 gingerbread biscuits for her grandchildren. She decorated them with raisins and nuts. First she decorated 15 cakes with raisins and then 15 cakes with nuts. At least how many cakes were decorated both with raisins and nuts?

- A) 4 B) 5 C) 6 D) 8 E) 10

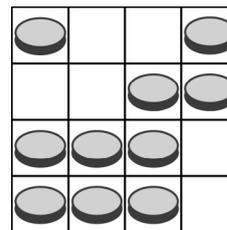
12. In a sudoku the numbers 1, 2, 3, 4 can occur only once in each column and in each row. In the mathematical sudoku below Patrick first writes in the results of the calculations. Then he completes the sudoku. Which number will Patrick put in the grey cell?

1 · 1		1 · 3	
	6 - 3		6 - 5
4 - 1	1 + 3		
9 - 7			

- A) 1 B) 2 C) 3 D) 4 E) 1 or 2

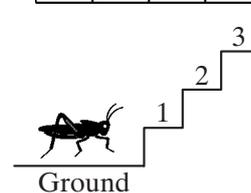
13. Among Nikolay's classmates there are twice as many girls as boys. Which of the following numbers can be equal to the number of all children in this class?  
 A) 30 B) 20 C) 24 D) 25 E) 29
14. In the animal's school, 3 kittens, 4 ducklings, 2 goslings and several lambs are taking lessons. The teacher owl found out that all of her pupils have 44 legs altogether. How many lambs are among them?  
 A) 6 B) 5 C) 4 D) 3 E) 2
15. There were a lot of fruits on the table: pears, apples, mandarines and plums, totally 496 fruits. Amount of pears was three times less than one of apples, amount of apples was five times less than one of mandarines, and amount of mandarines was seven times less than one of plumes. How many plums was there?  
 A) 350 B) 315 C) 455 D) 385 E) 420

16. There are coins on the board. We want to have 2 coins in each column and 2 coins in each row. How many coins need to be removed?  
 A) 0 B) 1 C) 2 D) 3 E) 4

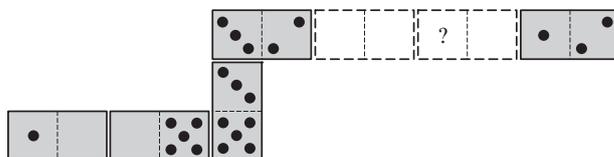


**5-point questions**

17. A grasshopper wants to climb a staircase with many steps. She makes only two different jumps: 3 steps up or 4 steps down. Beginning at the ground level, at least how many jumps will she have to make in order to take a rest on the 22th step?  
 A) 7 B) 9 C) 10 D) 12 E) 15



18. Frank made a domino snake of seven tiles. He put the tiles next to each other so that the sides with the same number of dots were touching. Originally the snake had 33 dots on its back. However, his brother George took away two tiles from the snake (see the picture). How many dots were in the place with the question mark?  
 A) 2 B) 3 C) 4 D) 5 E) 6

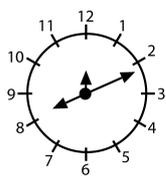
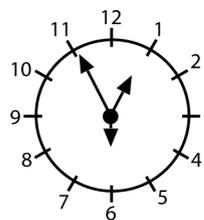


19. Gregor forms two numbers with the digits 1, 2, 3, 4, 5 and 6. Both numbers have three digits, each digit is used only once. He adds these two numbers. What is the greatest sum Gregor can get?  
 A) 975 B) 999 C) 1083 D) 1173 E) 1221

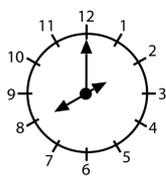
20. Laura, Iggy, Val and Kate want to be in one photo together. Kate and Laura are best friends and they want to stand next to each other. Iggy wants to stand next to Laura because he likes her. In how many possible ways can they arrange for the photo?

- A) 3 B) 4 C) 5 D) 6 E) 7

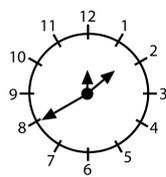
21. A special clock has 3 hands of different length (for hours, for minutes, and for seconds). We do not know which hand is which, but we know that the clock runs correctly. At 12:55:30 p.m. the hands were in position depicted on the right. How will this clock look like at 8:11:00 p.m.?



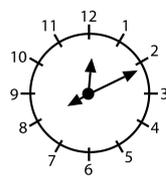
A)



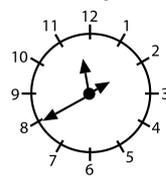
B)



C)



D)



E)

22. Michael chose some positive number, multiplied it by itself, added 1, multiplied the result by 10, added 3, and multiplied the result by 4. His final answer was 2012. What number did Michael choose?

- A) 11 B) 9 C) 8 D) 7 E) 5

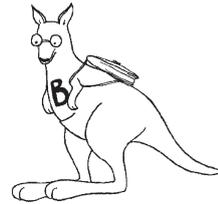
23. A rectangular paper sheet measures  $192 \times 84$  mm. You cut the sheet along just one straight line to get two parts, one of which is a square. Then you do the same with the non-square part of the sheet, and so on. What is the length of the side of the smallest square you can get with this procedure?

- A) 1 mm B) 4 mm C) 6 mm D) 10 mm E) 12 mm

24. In a soccer game the winner gains 3 points, while the loser gains 0 points. If the game is a draw, then the two teams gain 1 point each. A team has played 38 games gaining 80 points. Find the greatest possible number of games that the team lost.

- A) 12 B) 11 C) 10 D) 9 E) 8

## KANGAROO 2012



**Benjamin**  
**5 and 6 grades**

*Time allowed: 75 min*

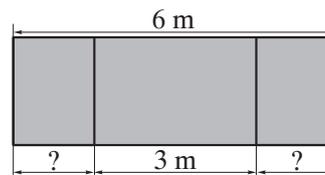
*Calculators are not permitted*

### 3-point questions

1. Basil wants to paint the slogan VIVAT KANGAROO on a wall. He wants different letters to be coloured differently, and the same letters to be coloured identically. How many colours will he need?

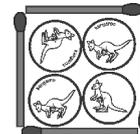
A) 7 B) 8 C) 9 D) 10 E) 13

2. A blackboard is 6 m wide. The width of the middle part is 3 m. The two other parts have equal width. How wide is the right-hand part?



A) 1 m B) 1,25 m C) 1,5 m D) 1,75 m E) 2 m

3. Sally can put 4 coins in a square built with 4 matches (see picture). At least how many matches will she need in order to build a square containing 16 coins that do not overlap?



A) 8 B) 10 C) 12 D) 15 E) 16

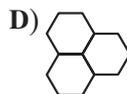
4. In a plane, the rows are numbered from 1 to 25, but there is no row number 13. Row number 15 has only 4 passenger seats, all the rest have 6 passenger seats. How many seats for passengers are there in the plane?

A) 120 B) 138 C) 142 D) 144 E) 150

5. When it is 4 o'clock in the afternoon in London, it is 5 o'clock in the afternoon in Madrid and it is 8 o'clock in the morning on the same day in San Francisco. Ann went to bed in San Francisco at 9 o'clock yesterday evening. What was the time in Madrid at that moment?

A) 6 o'clock yesterday morning B) 6 o'clock yesterday evening C) 12 o'clock yesterday afternoon D) 12 o'clock midnight E) 6 o'clock this morning

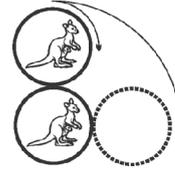
6. The picture shows a pattern of hexagons. We draw a new pattern by connecting all the midpoints of any neighbouring hexagons. What pattern do we get?



7. To the number 6 we add 3. Then we multiply the result by 2 and then we add 1. Then the final result will be the same as the result of the computation

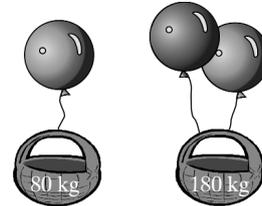
A)  $(6+3 \cdot 2)+1$  B)  $6+3 \cdot 2+1$  C)  $(6+3) \cdot (2+1)$  D)  $(6+3) \cdot 2+1$  E)  $6+3 \cdot (2+1)$

8. The upper coin is rotated without slipping around the fixed lower coin to a position shown on the picture. Which is the resulting relative position of kangaroos?



- A) B) C) D) E) Depends on the rotation speed

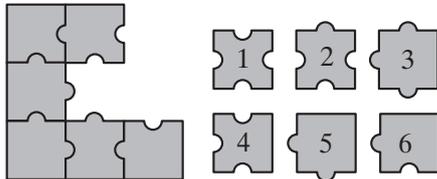
9. One balloon can lift a basket containing items weighing at most 80 kg. Two such balloons can lift the same basket containing items weighing at most 180 kg. What is the weight of the basket?



- A) 10 kg B) 20 kg C) 30 kg D) 40 kg E) 50 kg
10. Vivien and Mike were given some apples and pears by their grandmother. They had 25 pieces of fruit in their basket altogether. On the way home Vivien ate 1 apple and 3 pears, and Mike ate 3 apples and 2 pears. At home they found out that they brought home the same number of pears as apples. How many pears were they given by their grandmother?
- A) 12 B) 13 C) 16 D) 20 E) 21

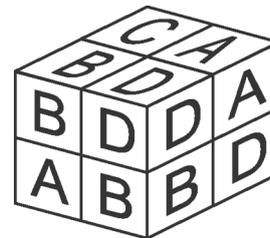
**4-point questions**

11. Which three of the numbered puzzle pieces should you add to the picture to complete the square?



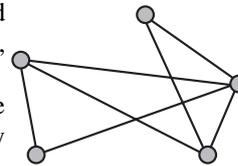
- A) 1, 3, 4 B) 1, 3, 6 C) 2, 3, 5 D) 2, 3, 6 E) 2, 5, 6

12. Lisa has 8 dice with the letters A, B, C and D, the same letter on all sides of each die. She builds a block with them. Two adjacent dice always have different letters. What letter is on the die that cannot be seen on the picture?



- A) A B) B C) C D) D E) E

13. There are five cities in Wonderland. Each pair of cities is connected by one road, either visible or invisible. On the map of Wonderland, there are only seven visible roads, as shown.



Alice has magical glasses: when she looks at the map through these glasses she only sees the roads that are otherwise invisible. How many invisible roads can she see?

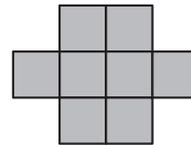
- A) 9 B) 8 C) 7 D) 3 E) 2

14. The positive integers have been coloured red, blue or green: 1 is red, 2 is blue, 3 is green, 4 is red, 5 is blue, 6 is green, and so on. Renate calculates the sum of a red number and a blue number. What colour can the resulting number be?

- A) Impossible to say B) Red or blue C) Only green D) Only red E) Only blue

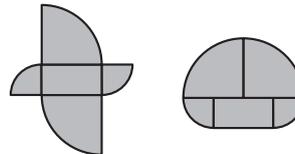
15. The perimeter of the figure below, built up of identical squares, is equal to 42 cm. What is the area of the figure?

- A)  $8 \text{ cm}^2$  B)  $9 \text{ cm}^2$  C)  $24 \text{ cm}^2$  D)  $72 \text{ cm}^2$  E)  $128 \text{ cm}^2$



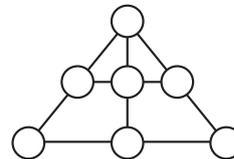
16. Look at the pictures. Both shapes are formed from the same five pieces. The rectangle measures  $5 \text{ cm} \times 10 \text{ cm}$ , and the other parts are quarters of two different circles. The difference between the perimeter lengths of the two shapes is

- A) 10 cm B) 15 cm C) 20 cm D) 25 cm E) 30 cm



17. Place the numbers from 1 to 7 in the circles, so that the sum of the numbers on each of the indicated lines of three circles is the same. What is the number at the top of the triangle?

- A) 1 B) 3 C) 4 D) 5 E) 6

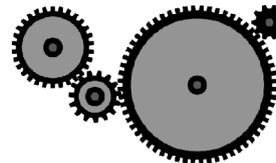


18. A rubber ball falls vertically through a height of 10 m from the roof of a house. After each impact on the ground it bounces back up to  $\frac{4}{5}$  of the previous height. How many times will the ball appear in front of a rectangular window whose bottom edge has a height of 5 m and whose top edge has a height of 6 m?

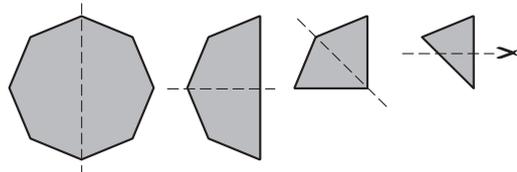
- A) 3 B) 4 C) 5 D) 6 E) 8

19. There are 4 gearwheels on fixed axes next to each other, as shown. The first one has 30 gears, the second one 15, the third one 60 and the last one 10. How many revolutions does the last gearwheel make, when the first one turns through one revolution?

- A) 3 B) 4 C) 6 D) 8 E) 9



20. A regular octagon is folded in half exactly three times until a triangle is obtained, as shown. Then the apex is cut off at right angles, as shown in the picture. If the paper is unfolded what will it look like?



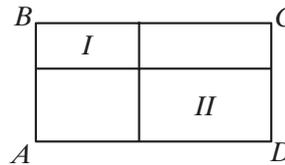
- A) B) C) D) E)

### 5-point questions

21. Winnie's vinegar-wine-water marinade contains vinegar and wine in the ratio 1 to 2, and wine and water in the ratio 3 to 1. Which of the following statements is true?

- A) There is more vinegar than wine  
 B) There is more wine than vinegar and water together  
 C) There is more vinegar than wine and water together  
 D) There is more water than vinegar and wine together  
 E) There is less vinegar than either water or wine

22. Rectangle  $ABCD$  is cut into four smaller rectangles, as shown in the figure. The perimeters of rectangles I and II equal 20 and 30, respectively. What is the perimeter of the original rectangle  $ABCD$ ?



A) 10 B) 50 C) 60 D) 80 E) 100

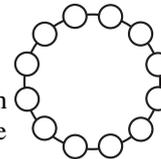
23. There were twelve children at a birthday party. Each child was either 4, 6, 7, 8 or 9 years old, with at least one child of each age. Four of them were 6 years old. In the group the most common age was 8 years old. What was the average age of the twelve children?

A) 5 B) 6 C) 7 D) 8 E) 9

24. The tango is danced in pairs, each consisting of one man and one woman. At a dance evening no more than 50 people are present. At one moment  $\frac{3}{4}$  of the men are dancing with  $\frac{4}{5}$  of the women. How many people are dancing at that moment?

A) 20 B) 24 C) 30 D) 32 E) 46

25. Kanga wants to arrange the twelve numbers from 1 to 12 in a circle such that any neighbouring numbers always differ by either 1 or 2. Which of the following pairs of numbers have to be neighbours?

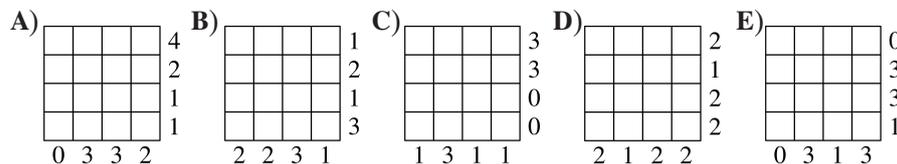


A) 5 and 6 B) 10 and 9 C) 6 and 7 D) 8 and 10 E) 4 and 3

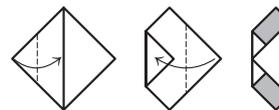
26. Peter wants to cut a rectangle of size  $6 \times 7$  into squares with integer sides. What is the minimal number of squares he can get?

A) 4 B) 5 C) 7 D) 9 E) 42

27. Some cells of the square table of size  $4 \times 4$  were colored red. The number of red cells in each row was indicated at the end of it, and the number of red cells in each column was indicated at the bottom of it. Then the red colour was eliminated. Which of the following tables can be the result?



28. A square-shaped piece of paper has area  $64 \text{ cm}^2$ . The square is folded twice as shown in the picture. What is the sum of the areas of the shaded rectangles?



A)  $10 \text{ cm}^2$  B)  $14 \text{ cm}^2$  C)  $15 \text{ cm}^2$  D)  $16 \text{ cm}^2$  E)  $24 \text{ cm}^2$

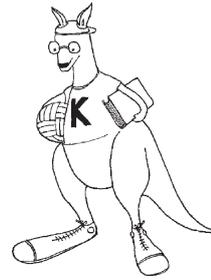
29. Abid's house number has 3 digits. Removing the first digit of Abid's house number, you obtain the house number of Ben. Removing the first digit of Ben's house number, you get the house number of Chiara. Adding the house numbers of Abid, Ben and Chiara gives 912. What is the second digit of Abid's house number?

A) 3 B) 4 C) 5 D) 6 E) 7

30. I give Ann and Bill two consecutive positive integers (for instance Ann 7 and Bill 6). They know their numbers are consecutive, they know their own number, but they do not know the number I gave to the other one. Then I heard the following discussion: Ann said to Bill: "I don't know your number". Bill said to Ann: "I don't know your number". Then Ann said to Bill: "Now I know your number! It is a divisor of 20.". What is Ann's number?

A) 2 B) 3 C) 4 D) 5 E) 6

## KANGAROO 2012



**Cadet**  
**7 and 8 grades**

*Time allowed: 75 min*

*Calculators are not permitted*

### 3-point questions

1. Four chocolate bars cost 6 EUR more than one chocolate bar. What is the cost of one chocolate bar?

A) 1 EUR B) 2 EUR C) 3 EUR D) 4 EUR E) 1,5 EUR

2.  $11,11 - 1,111 =$

A) 9,009 B) 9,0909 C) 9,99 D) 9,999 E) 10

3. A watch is placed face up on a table so that its minute hand points east. How many minutes pass before the minute hand points north for the first time?

A) 45 min B) 40 min C) 30 min D) 20 min E) 15 min

4. Mary has a pair of scissors and five cardboard letters. She cuts each letter exactly once (along a straight line) so that it falls apart in as many pieces as possible. Which letter falls apart into the most pieces?

A) O B) F C) S D) H E) M

5. A dragon has five heads. Every time a head is chopped off, five new heads grow. If six heads are chopped off one by one, how many heads will the dragon finally have?

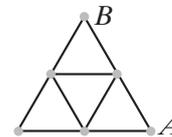
A) 25 B) 28 C) 29 D) 30 E) 35

6. In which of the following expressions can we replace each occurrence of the number 8 by the same positive number (other than 8) and obtain the same result?

A)  $(8+8) : 8+8$  B)  $8 \cdot (8+8) : 8$  C)  $8+8-8+8$  D)  $(8+8-8) \cdot 8$  E)  $(8+8-8) : 8$

7. Each of the nine paths in a park is 100 m long. Ann wants to go from A to B without going along any path more than once. What is the length (in meters) of the longest route she can choose?

A) 900 B) 800 C) 700 D) 600 E) 400

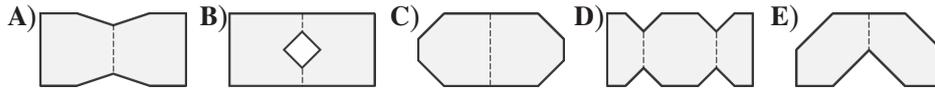


8. The diagram shows two triangles. In how many ways can you choose two vertices, one in each triangle, so that the straight line through the vertices does not cross either triangle?

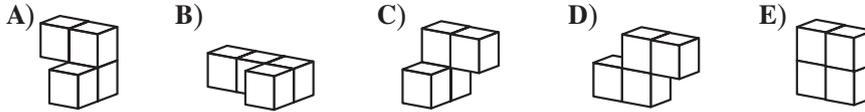
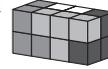
A) 1 B) 2 C) 3 D) 4 E) More than 4



9. Werner folds a sheet of paper as shown in the figure and makes two straight cuts with a pair of scissors. He then opens up the paper again. Which of the following shapes cannot be the result?



10. A cuboid is made of four pieces, as shown. Each piece consists of four cubes and is a single colour. What is the shape of the white piece?



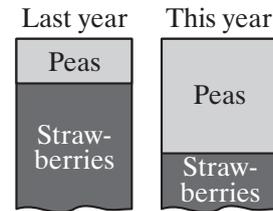
#### 4-point questions

11. Kanga forms two 4-digit natural numbers using each of the digits 1, 2, 3, 4, 5, 6, 7 and 8 exactly once. Kanga wants the sum of the two numbers to be as small as possible. What is the value of this smallest possible sum?

A) 2468 B) 3333 C) 3825 D) 4734 E) 6918

12. Mrs Gardner grows peas and strawberries. This year she has changed the rectangular pea bed to a square by lengthening one of its sides by 3 metres. As a result of this change, the area of the strawberry bed was reduced by  $15 \text{ m}^2$ . What was the area of the pea bed before the change?

A)  $5 \text{ m}^2$  B)  $9 \text{ m}^2$  C)  $10 \text{ m}^2$  D)  $15 \text{ m}^2$  E)  $18 \text{ m}^2$



13. Barbara wants to complete the diagram 

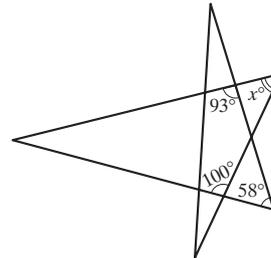
10				130
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 by inserting three numbers, one in each empty cell. She wants the sum of the first three numbers to be 100, the sum of the three middle numbers to be 200 and the sum of the last three numbers to be 300. What number should Barbara insert in the middle cell of the diagram?

A) 50 B) 60 C) 70 D) 75 E) 100

14. In the figure, what is the value of  $x$ ?

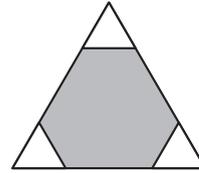
A)  $35^\circ$  B)  $42^\circ$  C)  $51^\circ$  D)  $65^\circ$  E)  $109^\circ$



15. Four cards each have a number written on one side and a phrase written on the other. The four phrases are “divisible by 7”, “prime”, “odd” and “greater than 100”, and the four numbers are 2, 5, 7 and 12. On each card, the number does not correspond to the phrase on the other side. What number is written on the same card as the phrase “greater than 100”?

A) 2 B) 5 C) 7 D) 12 E) Impossible to determine

16. Three small equilateral triangles of the same size are cut from the corners of a larger equilateral triangle with sides of 6 cm, as shown. The sum of the perimeters of the three small triangles is equal to the perimeter of the remaining grey hexagon. What is the side length of the small triangles?

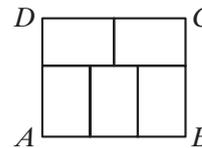


A) 1 cm B) 1,2 cm C) 1,25 cm D) 1,5 cm E) 2 cm

17. A piece of cheese is cut into a large number of pieces. During the course of the day, a number of mice came and stole some pieces, watched by the lazy cat Ginger. Ginger noticed that each mouse stole a different number of pieces less than 10, and that no mouse stole exactly twice as many pieces as any other mouse. What is the largest number of mice that Ginger could have seen stealing cheese?

A) 4 B) 5 C) 6 D) 7 E) 8

18. Rectangle  $ABCD$  is divided into five identical rectangles (see pic.) of 20 cm perimeter each. What is the area of the rectangle  $ABCD$ ?

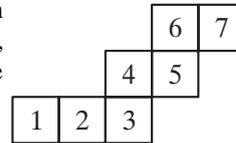


A)  $72 \text{ cm}^2$  B)  $112 \text{ cm}^2$  C)  $120 \text{ cm}^2$  D)  $140 \text{ cm}^2$  E)  $150 \text{ cm}^2$

19. A magical talking square originally has sides of length 8 cm. If he tells the truth, then his sides become 2 cm shorter. If he lies, then his perimeter doubles. He makes four statements, two true and two false, in some order. What is the largest possible perimeter of the square after the four statements?

A) 28 cm B) 80 cm C) 88 cm D) 112 cm E) 120 cm

20. A cube is rolled on a plane so that it turns around its edges. Its bottom face passes through the positions 1, 2, 3, 4, 5, 6, and 7 in that order, as shown. Which two of these positions were occupied by the same face of the cube?



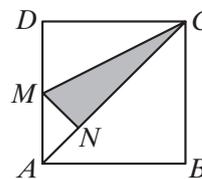
A) 1 and 7 B) 1 and 6 C) 1 and 5 D) 2 and 7 E) 2 and 6

### 5-point questions

21. Rick has five cubes. When he arranges them from smallest to largest, the difference between the heights of any two neighbouring cubes is 2 cm. The largest cube is as high as a tower built from the two smallest cubes. How high is a tower built from all five cubes?

A) 6 cm B) 14 cm C) 22 cm D) 44 cm E) 50 cm

22. In the diagram  $ABCD$  is a square,  $M$  is the midpoint of  $AD$  and  $MN$  is perpendicular to  $AC$ . What is the ratio of the area of the shaded triangle  $MNC$  to the area of the square?

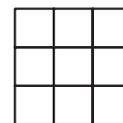
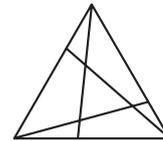


A) 1 : 6 B) 1 : 5 C) 7 : 36 D) 3 : 16 E) 7 : 40

23. The tango is danced in pairs, each consisting of one man and one woman. At a dance evening no more than 50 people are present. At one moment  $\frac{3}{4}$  of the men are dancing with  $\frac{4}{5}$  of the women. How many people are dancing at that moment?

A) 20 B) 24 C) 30 D) 32 E) 46

24. David wants to arrange the twelve numbers from 1 to 12 in a circle so that any two neighbouring numbers differ by either 2 or 3. Which of the following pairs of numbers have to be neighbours?  
 A) 5 and 8   B) 3 and 5   C) 7 and 9   D) 6 and 8   E) 4 and 6
25. Some three-digit integers have the following property: if you remove the first digit of the number, you get a perfect square; if instead you remove the last digit of the number, you also get a perfect square. What is the sum of all the three-digit integers with this curious property?  
 A) 1013   B) 1177   C) 1465   D) 1993   E) 2016
26. A book contains 30 stories, each starting on a new page. The lengths of the stories are 1, 2, 3, ..., 30 pages. The first story starts on the first page. What is the largest number of stories that can start on an odd-numbered page?  
 A) 15   B) 18   C) 20   D) 21   E) 23
27. An equilateral triangle starts in a given position and is moved to new positions in a sequence of steps. At each step it is rotated about its centre, first by  $3^\circ$ , then by a further  $9^\circ$ , then by a further  $27^\circ$ , and so on (at the  $n$ -th step it is rotated by a further  $(3^n)^\circ$ ). How many different positions, including the initial position, will the triangle occupy? Two positions are considered equal if the triangle covers the same part of the plane.  
 A) 3   B) 4   C) 5   D) 6   E) 360
28. A rope is folded in half, then in half again, and then in half again. Finally the folded rope is cut through, forming several strands. The lengths of two of the strands are 4 m and 9 m. Which of the following could not have been the length of the whole rope?  
 A) 52 m   B) 68 m   C) 72 m   D) 88 m   E) All the previous are possible
29. A triangle is divided into four triangles and three quadrilaterals by three straight line segments. The sum of the perimeters of the three quadrilaterals is equal to 25 cm. The sum of the perimeters of the four triangles is equal to 20 cm. The perimeter of the whole triangle is equal to 19 cm. What is the sum of the lengths of the three straight line segments?  
 A) 11 cm   B) 12 cm   C) 13 cm   D) 15 cm   E) 16 cm
30. A positive number is to be placed in each cell of the  $3 \times 3$  grid shown, so that: in each row and each column, the product of the three numbers is equal to 1; and in each  $2 \times 2$  square, the product of the four numbers is equal to 2. What number should be placed in the central cell?  
 A) 16   B) 8   C) 4   D)  $\frac{1}{4}$    E)  $\frac{1}{8}$



## KANGAROO 2012

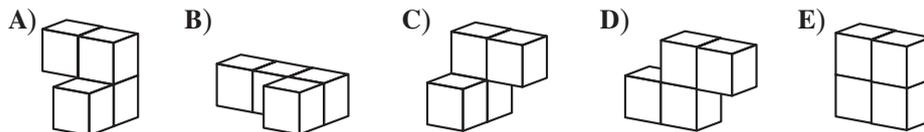


Time allowed: 75 min  
 Calculators are not permitted

### 3-point questions

Junior  
 9 and 10 grades

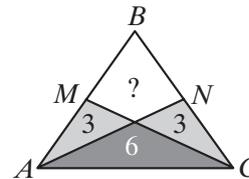
1. A cuboid is made of four pieces, as shown. Each piece consists of four cubes and is a single colour. What is the shape of the white piece?



2.  $11.11 - 1.111 =$

A) 9.009 B) 9.0909 C) 9.99 D) 9.999 E) 10

3. The diagram shows an isosceles triangle;  $M$  and  $N$  are the midpoints of the equal sides. The triangle has been divided into four regions by two straight lines. Three of the regions have areas 3, 3 and 6, as shown. What is the area of the fourth region?

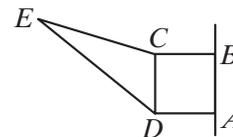


A) 3 B) 4 C) 5 D) 6 E) 7

4. When Alice wants to send a message to Bob, she uses the following system, known to Bob. For each letter in the message, she converts the letter to a number, using  $A = 1$ ,  $B = 2$ ,  $C = 3$ , ...,  $Z = 26$ , and then calculates  $2 \times \text{number} + 9$ . Alice sends the sequence of results to Bob. This morning Bob received the sequence  $25-19-45-38$ . What was the original message?

A) HERO B) HELP C) HEAR D) HERS E) Alice has made a mistake

5. The square  $ABCD$  has side length 4 cm. The square has the same area as the triangle  $ECD$ . What is the distance from the point  $E$  to the line  $AB$ ?



A) 8 cm B)  $(4 + 2\sqrt{3})$  cm C) 12 cm D)  $10\sqrt{2}$  cm  
 E) Depends on the location of  $E$

6. The sum of the digits of a seven-digit integer is 6. What is the product of these digits?

A) 0 B) 6 C) 7 D)  $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7$  E) 5

7.  $ABC$  is a right-angled triangle whose legs are 6 cm and 8 cm long. The points  $K$ ,  $L$ ,  $M$  are the midpoints of the sides of the triangle. How long is the perimeter of the triangle  $KLM$ ?

A) 10 cm B) 12 cm C) 15 cm D) 20 cm E) 24 cm

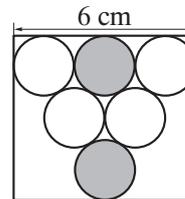
8. In four of the following expressions we can replace each number 8 by another positive number (always using the same number for every replacement) and obtain the same result. Which expression does not have this property?

A)  $(8+8-8) : 8$  B)  $8+(8:8)-8$  C)  $8:(8+8+8)$  D)  $8-(8:8)+8$  E)  $8 \cdot (8:8) : 8$

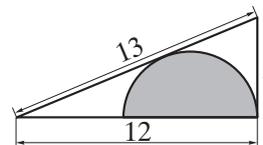
9. Two sides of a quadrilateral are equal to 1 and 4. One of the diagonals has length 2, and divides the quadrilateral into two isosceles triangles. How long is the perimeter of the quadrilateral?  
 A) 8 B) 9 C) 10 D) 11 E) 12
10. Each of the numbers 144 and 220 is divided by the positive integer  $N$ , giving a remainder of 11 in each case. What is the value of  $N$ ?  
 A) 7 B) 11 C) 15 D) 19 E) 38

**4-point questions**

11. If Adam stands on the table and Mike stands on the floor, then Adam is 80 cm taller than Mike. If Mike stands on the same table and Adam is on the floor, then Mike is one metre taller than Adam. How high is the table?  
 A) 20 cm B) 80 cm C) 90 cm D) 100 cm E) 120 cm
12. Denis and Mary were tossing a coin. If the coin showed heads the winner was Mary and Denis had to give her 2 candies. If the coin showed tails the winner was Denis and Mary had to give him three candies. After 30 games each of them had as many candies as at the start of the game. How many times did Denis win?  
 A) 6 B) 12 C) 18 D) 24 E) 30
13. A rectangle of length 6 cm encloses an “equilateral triangle” of touching circles, as shown. What is the shortest distance between the two grey circles?  
 A) 1 cm B)  $\sqrt{2}$  cm C)  $(2\sqrt{3} - 2)$  cm D)  $\frac{\pi}{2}$  cm E) 2 cm



14. In Billy’s room there are four clocks. Each clock is either slow or fast. The first clock is wrong by 2 minutes, the second clock by 3 minutes, the third by 4 minutes and the fourth by 5 minutes. One day Billy wanted to know the exact time by his clocks, which read 6 minutes to 3, 3 minutes to 3, 2 minutes past 3, and 3 minutes past 3. What was the exact time then?  
 A) 3:00 B) 2:57 C) 2:58 D) 2:59 E) 3:01
15. The diagram shows a right triangle with sides 5, 12 and 13. What is the radius of the inscribed semicircle?  
 A)  $\frac{7}{3}$  B)  $\frac{10}{3}$  C)  $\frac{12}{3}$  D)  $\frac{13}{3}$  E)  $\frac{17}{3}$

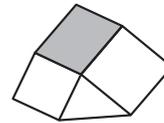


16. How many four-digit integers are there for which the hundreds digit is 3 and the sum of the other three digits is also 3?  
 A) 2 B) 3 C) 4 D) 5 E) 6
17. Kanga is writing twelve numbers chosen from 1 to 9 in the cells of a grid, so that the sum of every row is the same and the sum of every column is the same. Kanga has already written some of the numbers, as shown. What number should be written in the shaded square?  
 A) 1 B) 4 C) 6 D) 8 E) 9

2	4		2
	3	3	
6		1	

18. Three athletes Kan, Ga and Roo took part in a Marathon race. Before the race, four spectators discussed the athletes' chances. The first said: "Either Kan or Ga will win". The second said: "If Ga is the second, then Roo will win". The third said: "If Ga is the third, then Kan will not win". The fourth said: "Either Ga or Roo will be the second". After the race it turned out that all four statements were true. Kan, Ga and Roo were the three top athletes in the race. In what order did they finish?
- A) Kan, Ga, Roo B) Kan, Roo, Ga C) Roo, Ga, Kan D) Ga, Roo, Kan E) Ga, Kan, Roo

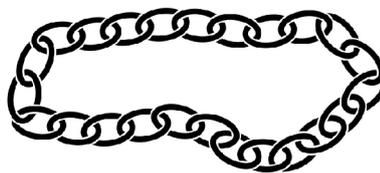
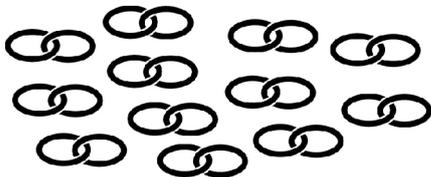
19. The diagram shows a shape formed from two squares with sides 4 and 5, a triangle with area 8 and a shaded parallelogram. What is the area of the parallelogram?
- A) 15 B) 16 C) 18 D) 20 E) 21



20. Ann has written  $2012 = m^m \cdot (m^k - k)$  for some positive integer values of  $m$  and  $k$ . What is the value of  $k$ ?
- A) 2 B) 3 C) 4 D) 9 E) 11

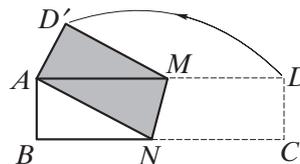
**5-point questions**

21. A jeweller has 12 pieces of chain, each with two links. He wants to make one big closed necklace of them, as shown. To do this he has to open some links (and close them afterwards). What is the smallest number of links he has to open?



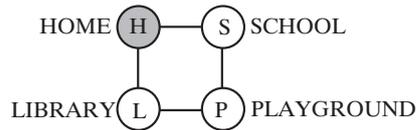
- A) 8 B) 9 C) 10 D) 11 E) 12

22. A rectangular piece of paper  $ABCD$  measuring  $4 \text{ cm} \times 16 \text{ cm}$  is folded along the line  $MN$  so that vertex  $C$  coincides with vertex  $A$ , as shown. What is the area of the pentagon  $ABNMD'$  in  $\text{cm}^2$ ?
- A) 17 B) 27 C) 37 D) 47 E) 57



23. Train G passes a milestone in 8 seconds before meeting train H. The two trains pass each other in 9 seconds. Then train H passes the milestone in 12 seconds. If both trains G and H go with constant speeds, what is the ratio of their lengths equal to?
- A) 2 B) 1 C)  $\frac{2}{3}$  D)  $\frac{1}{2}$  E)  $\frac{3}{2}$
24. The last non-zero digit of the number  $K = 2^{59} \cdot 3^4 \cdot 5^{53}$  is
- A) 1 B) 2 C) 4 D) 6 E) 9

25. Peter creates a Kangaroo game. The diagram shows the board for the game. At the start, the Kangaroo is at the School  $S$ . According to the rules of the game, from any position except Home  $H$  the Kangaroo can jump to either of the two neighboring positions. When the Kangaroo lands on  $H$  the game is over. In how many ways can the Kangaroo move from  $S$  to  $H$  in exactly 13 jumps?  
 A) 12 B) 32 C) 64 D) 144 E) 1024

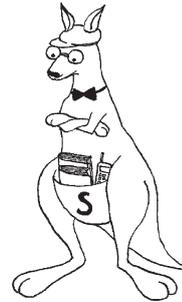


26. You are given 5 lamps, each of which can be switched to “on” or “off”. Each time you switch any lamp, you change its status; moreover, the status of exactly one other randomly chosen lamp is also changed. (For the same lamp, the choice of the other lamp may be different each time.) At the beginning, all the lamps are off. Then you make 10 such switch operations. Which of the following statements is now true?  
 A) It is impossible for all the lamps to be off B) All the lamps are definitely on  
 C) It is impossible for all the lamps to be on D) All the lamps are definitely off  
 E) None of the statements A to D is correct
27. Six different positive integers are given, the biggest of them being  $n$ . There exists exactly one pair of these integers such that the smaller number does not divide the bigger one. What is the smallest possible value of  $n$ ?  
 A) 18 B) 20 C) 24 D) 36 E) 45
28. Nick wrote down all three-digit integers and for each of them he wrote down the product of its digits. After that Nick found the sum of all these products. What total should Nick obtain?  
 A) 45 B)  $45^2$  C)  $45^3$  D)  $2^{45}$  E)  $3^{45}$
29. The numbers from 1 to 120 have been written into 15 rows, in the manner indicated in the diagram. For which column (counting from the left) is the sum of the numbers the largest?

1							...	
2	3						...	
4	5	6					...	
7	8	9	10				...	
11	12	13	14	15			...	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
106	107	108	109	110	111	112	...	120

- A) 1 B) 5 C) 7 D) 10 E) 13
30. Let  $A, B, C, D, E, F, G, H$  be the eight vertices of a convex octagon, taken in order. Choose a vertex from  $C, D, E, F, G, H$  and draw the line segment connecting it with vertex  $A$ . Once more, choose a vertex from the same six vertices, but now draw the line segment connecting it with vertex  $B$ . The segments divide the octagon into exactly three regions. How many such divisions are there?  
 A) 6 B) 9 C) 10 D) 12 E) 16

## KANGAROO 2012

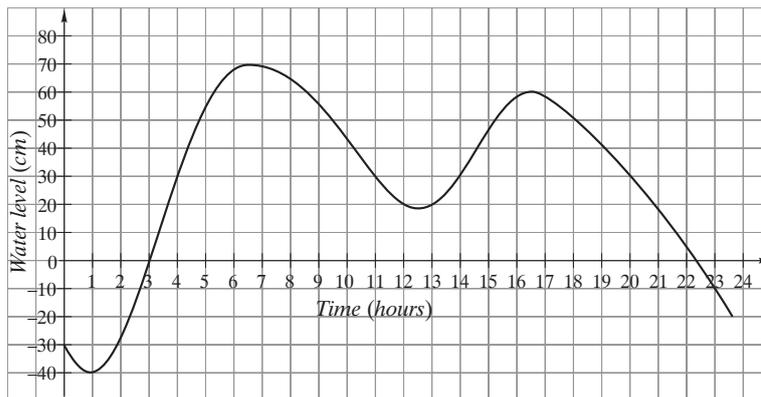


Time allowed: 75 min  
 Calculators are not permitted

**Student**  
**11 and 12 grades**

### 3-point questions

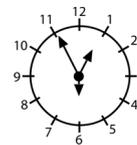
1. The water level in a port city rises and falls on a certain day as shown in the figure. For how many hours was the water level above 30 cm on that day?



- A) 5 B) 6 C) 7 D) 9 E) 13
2. The number  $\sqrt[3]{2\sqrt{2}}$  is equal to  
 A) 1 B)  $\sqrt{2}$  C)  $\sqrt[6]{4}$  D)  $\sqrt[3]{4}$  E) 2
3. In a list of five numbers 

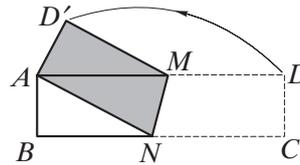
2				12
---	--	--	--	----

 the first number is 2 and the last number is 12. The product of the first three numbers is 30, the product of the three in the middle is 90 and the product of the last three numbers is 360. Which number is in the centre of the list?  
 A) 3 B) 4 C) 5 D) 6 E) 10
4. A clock has 3 hands of different length (for hours, for minutes, and for seconds). We do not know which hand is which, but we know that the clock runs correctly. At 12:55:30 the hands were in the positions shown. Which of the pictures shows this clock at 8:10:00?



- A) B) C) D) E)

5. A rectangular piece of paper  $ABCD$  measuring  $16\text{ cm} \times 4\text{ cm}$  is folded along the line  $MN$  so that vertex  $C$  coincides with vertex  $A$ , as shown in the picture. What is the area of quadrilateral  $ANMD'$ ?



A)  $28\text{ cm}^2$  B)  $30\text{ cm}^2$  C)  $32\text{ cm}^2$  D)  $48\text{ cm}^2$  E)  $56\text{ cm}^2$

6. The sum of the digits of a nine-digit integer is 8. What is the product of these digits?

A) 0 B) 1 C) 8 D) 9 E)  $9!$

7. The maximum value of the integer  $n$ , for which  $n^{200} < 5^{300}$ , is equal to:

A) 5 B) 6 C) 8 D) 11 E) 12

8. Which of the following functions satisfies the equation  $f\left(\frac{1}{x}\right) = \frac{1}{f(x)}$ ?

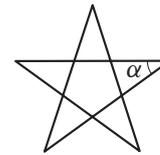
A)  $f(x) = \frac{2}{x}$  B)  $f(x) = \frac{1}{x+1}$  C)  $f(x) = 1 + \frac{1}{x}$  D)  $f(x) = \frac{1}{x}$  E)  $f(x) = x + \frac{1}{x}$

9. A real number  $x$  satisfies  $x^3 < 64 < x^2$ . Which statement is correct?

A)  $0 < x < 64$  B)  $-8 < x < 4$  C)  $x > 8$  D)  $-4 < x < 8$  E)  $x < -8$

10. What is the size of the angle  $\alpha$  in the regular 5-pointed star?

A)  $24^\circ$  B)  $30^\circ$  C)  $36^\circ$  D)  $45^\circ$  E)  $72^\circ$



#### 4-point questions

11. My age is a two-digit integer, which is a power of 5, and my cousin's age is a two-digit integer, which is a power of 2. The sum of the digits of our ages is an odd number. What is the product of the digits of our ages?

A) 240 B) 2010 C) 60 D) 50 E) 300

12. A travel agency organized four optional tours of Sicily for a group of tourists. Each tour had a participation rate of 80%. What is the smallest possible percentage of tourists who took part in all four tours?

A) 80% B) 60% C) 40% D) 20% E) 16%

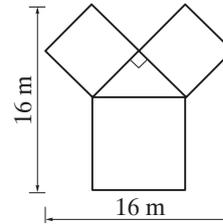
13. The set of solutions for the inequality  $|x| + |x - 3| > 3$  is:

A)  $(-\infty; 0) \cup (3; +\infty)$  B)  $(-3; 3)$  C)  $(-\infty; -3)$  D)  $(-3; +\infty)$  E)  $(-\infty; +\infty)$

14. School marks in Slovakia are divided into five degrees, from 1 (the best) to 5. In one Slovak school, a test didn't turn out very well in the 4th class. The average mark was 4. Boys did a little better, their average mark was 3.6 while the average mark of the girls was 4.2. Which of the following statements about the class is correct?

A) There are twice as many boys as girls  
 B) There are 4 times as many boys as girls  
 C) There are twice as many girls as boys  
 D) There are 4 times as many girls as boys  
 E) There are as many boys as girls

15. The picture shows the plan of a rose bed. White roses grow in the two equal squares, and red roses grow in the third square. Yellow roses grow in the right-angled triangle. Both the length and width of the bed are 16 m. What is the area of the rose bed?

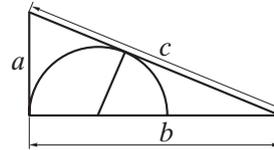


- A)  $114 \text{ m}^2$  B)  $130 \text{ m}^2$  C)  $144 \text{ m}^2$  D)  $160 \text{ m}^2$  E)  $186 \text{ m}^2$

16. All the tickets for the first row in a cinema were sold. The seats are numbered consecutively starting with 1. An extra ticket was sold for one seat by mistake. The sum of the seat numbers on all tickets sold for that row is equal to 857. What is the number of the seat for which two tickets were sold?

- A) 4 B) 16 C) 25 D) 37 E) 42

17. We are given a right-angled triangle with sides of length  $a$ ,  $b$  and  $c$ . What is the radius  $r$  of the inscribed semicircle shown in the figure?

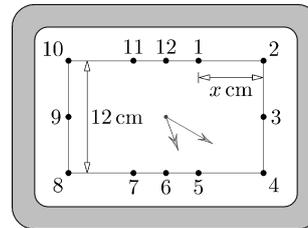


- A)  $\frac{a(c-a)}{2b}$  B)  $\frac{ab}{a+b+c}$  C)  $\frac{ab}{b+c}$  D)  $\frac{2ab}{a+b+c}$  E)  $\frac{ab}{a+c}$

18. A square  $ABCD$  has sides of length 2.  $E$  and  $F$  are the midpoints of the sides  $AB$  and  $AD$  respectively.  $G$  is a point on the segment  $CF$  such that  $3CG = 2GF$ . The area of triangle  $BEG$  is:

- A)  $\frac{7}{10}$  B)  $\frac{4}{5}$  C)  $\frac{8}{5}$  D)  $\frac{3}{5}$  E)  $\frac{6}{5}$

19. The clock in the picture is rectangular in shape, but each hand moves at a constant rate, like a normal clock. The distance between the numbers 8 and 10 on the dial is 12 cm and the distance between 1 and 2 is  $x$  cm. What is the value of  $x$ ?



- A)  $3\sqrt{3}$  B)  $2\sqrt{3}$  C)  $4\sqrt{3}$  D)  $2 + \sqrt{3}$  E)  $12 - 3\sqrt{3}$

20. A kangaroo wants to build a row of standard dice (for a standard die, each pair of opposite faces has a total of 7 dots). He can glue two faces together if they have the same number of dots. He would like the total number of dots on the outer faces of the dice in the row to be 2012. How many dice does he need?

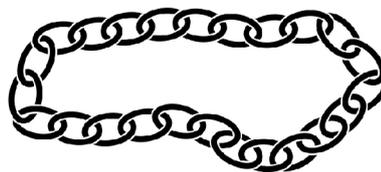
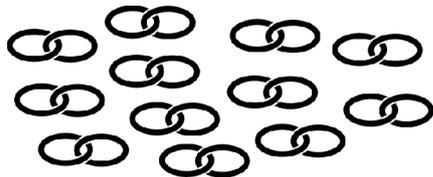
- A) 70 B) 71 C) 142 D) 143 E) A total of 2012 dots is impossible

### 5-point questions

21. What is the smallest possible size of an angle in an isosceles triangle  $ABC$  that has a median that divides the triangle into two isosceles triangles?

- A)  $15^\circ$  B)  $22,5^\circ$  C)  $30^\circ$  D)  $36^\circ$  E)  $45^\circ$

22. A jeweller has 12 pieces of chain, each with two links. He wants to make one big closed necklace of them, as shown. To do this he has to open some links (and close them afterwards). What is the smallest number of links he has to open?

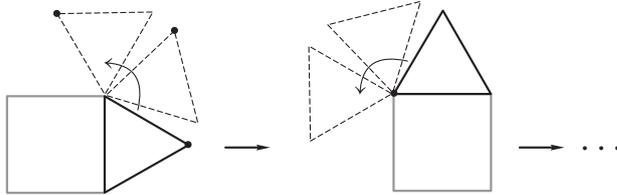


- A) 8 B) 9 C) 10 D) 11 E) 12

23. Consider two operations which can be performed on a fraction: 1) increase the numerator by 8; 2) increase the denominator by 7. Having performed a total number of  $n$  such operations in some order, starting with the fraction  $\frac{7}{8}$  we obtain a fraction of equal value. What is the smallest possible value of  $n$ ?

A) 56 B) 81 C) 109 D) 113 E) This is impossible

24. An equilateral triangle rolls without slipping around a square with side length 1 (see picture).



What is the length of the path that the marked point covers until the triangle and the point reach their starting positions the next time?

A)  $4\pi$  B)  $\frac{28}{3}\pi$  C)  $8\pi$  D)  $\frac{14}{3}\pi$  E)  $\frac{21}{2}\pi$

25. How many permutations  $(x_1, x_2, x_3, x_4)$  of the set of integers  $\{1, 2, 3, 4\}$  have the property that the sum  $x_1x_2 + x_2x_3 + x_3x_4 + x_4x_1$  is divisible by 3?

A) 8 B) 12 C) 14 D) 16 E) 24

26. After an algebra lesson, the following was left on the blackboard: the graph of the function  $y = x^2$  and 2012 lines parallel to the line  $y = x$ , each of which intersects the parabola in two points. The sum of the  $x$ -coordinates of the points of intersection of the lines and the parabola is:

A) 0 B) 1 C) 1006 D) 2012 E) Impossible to determine

27. Three vertices of a cube are  $P(3; 4; 1)$ ,  $Q(5; 2; 9)$  and  $R(1; 6; 5)$ . Which point is the centre of the cube?

A)  $A(4; 3; 5)$  B)  $B(2; 5; 3)$  C)  $C(3; 4; 7)$  D)  $D(3; 4; 5)$  E)  $E(2; 3; 5)$

28. In the sequence 1, 1, 0, 1,  $-1, \dots$ , each of the first two terms  $a_1$  and  $a_2$  is 1. The third term is the difference of the preceding two terms, that is,  $a_3 = a_1 - a_2$ . The fourth is the sum of the two preceding terms, that is,  $a_4 = a_2 + a_3$ . Then  $a_5 = a_3 - a_4$ ,  $a_6 = a_4 + a_5$ , and so on. What is the sum of the first 100 terms of this sequence?

A) 0 B) 3 C)  $-21$  D) 100 E)  $-1$

29. Ioana picks out two numbers  $a$  and  $b$  from the set  $\{1, 2, 3, \dots, 26\}$ . The product  $ab$  is equal to the sum of the remaining 24 numbers. What is the value of  $|a - b|$ ?

A) 10 B) 9 C) 7 D) 2 E) 6

30. Every cat in Wonderland is either wise or mad. If a wise cat happens to be in one room with 3 mad ones it turns mad. If a mad cat happens to be in one room with 3 wise ones it is exposed by them as mad. Three cats entered an empty room. Soon after the 4th cat entered, the 1st one went out. After the 5th cat entered, the 2nd one went out, etc. After the 2012th cat entered, it happened for the first time that one of the cats was exposed as mad. Which of these cats could both have been mad after entering the room?

A) The 1st one and the 2011th one B) The 2nd one and the 2010th one  
C) The 3rd one and the 2009th one D) The 4th one and the last one  
E) The 2nd one and the 2011th one